To the Bookſeller.

S I R, *London, Feb.* 27. 1726.

I Have peruſed ſo much of this Book of Brigadeer General *Douglaſs's*, as was neceſſary to my paſſing ſome Judgment upon his Deſign and Performance. They ſeem to me to be New, True, and Uſeful. He taking his Angles, and cloſing his Figures by eaſy Obſervations in the Field, and the Uſe of a ſingle Parallelogram at Home; without any Theodolite or Semicircle, or any Knowledge of the Number of Degrees his ſeveral Angles contain; which is always neceſſary, but always very difficult in the common way. And what greatly recommends this Method is this, that when the Author was imployed under the Baron *de Coehorn*, that great Engineer ſaw and approved of this Method. And though there be no Teleſcopick Sights here ordered to be applied to this Inſtrument, yet is it equally capable of them, as are any other Inſtruments of the ſame Nature. I think therefore your Publication of this Book will be acceptable to the Lovers and Practitioners in this Part of the Mathematicks. This is the Opinion of

Your Humble Servant

Will. Whiſton,

Sometime Profeſſor of the Mathematicks in the Univerſity of *Cambridge*.

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The Surveyor's Utmost Desire Fulfilled:

OR, THE

A R T

OF

PLANOMETRY, LONGEMETRY, AND ALTEMETRY,

Brought to its greatest Perfection, by the Help of the Ungraduated Instrument, called the *INFALLIBLE*.

LONDON:
Printed for JOHN OSBORN, and THOMAS LONGMAN, at the Ship in *Pater-noster-Row*,
M.DCC.XXVII.

TO

His Grace

JAMES,

Duke of *Queensberry* and *Dover*,

One of the Lords of His Majesty's Bed-Chamber, &c.

THE Transcendency of your Knowledge in all liberal Arts and Sciences, as well as in States Government, raised my Thoughts towards finding out some Return worthy of your

Lordſhip's Attention, for the bountiful Dews of Plenty wherewith I have been ſprinkled under this moſt flouriſhing Government, ever ſince the happy Acceſſion of our Glorious Monarch King *GEORGE*, to the Imperial Crown of theſe Realms.

Firſt I took a View of the ſublime Parts of the Mathematicks, but found the incomparable Sir *Iſaac Newton*, with *Gregory the Great*, had been there, and had ſo marked all thoſe Rarities with their proper Names and Seals, that if I ſhould have pick'd any Thing out of thoſe ſelect Bundles, it would have been look'd upon as a ſtol'n Work, or as a Botch, mangling the Works of a great Artiſt.

So humbly ſcrambling amongſt the Rubbiſh of theſe Reverſions, found as if it were a rough Diamond, which (in the Wars of King *William* and Queen *Anne*, both of Glorious Memory) I poliſh'd,

liſh'd, and put ſuch a Cut upon it, that the renowned Baron *de Coehorn*, (whom I had the Honour to ſerve under in the augmenting and repairing the Fortifications of *Maſtrick*, and the *Grave*, as well as in ſeveral Sieges, *&c.*) nam'd this my Invention, The *INFALLIBLE*; giving for Reaſon to all our Engineers, that the Practice with this ſimple Inſtrument, being founded upon the infallible Axioms of the Mathematicks, it could not but be obvious to the meaneſt Capacity therewith to perform all Operations in *Altemetry*, *Longemetry*, and *Planometry*, without the leaſt Error; which the greateſt Artiſt in *Europe* cannot promiſe to do, with any other Geometrical Inſtrument hitherto invented.

I ſhall then, for this ſmall Mite, which no other can lay Claim to, moſt humbly beg your Grace's Acceptance and Protection for the Author; who, big with the lively Idea's of the noble and

memorable Actions of your most Illustrious Ancestors, as well as with that of your own personal Splendour, and outshining Brightness, is ready, as it were, to break forth in the greatest Raptures of due Praise. But your Lordship's unparallell'd Modesty and Aversion to all Flattery, at once stops my Mouth, being sensible, that after saying all I could on that most copious and glorious Subject, I should but come vastly short of your Due.

I shall therefore conclude: most humbly begging your Grace would be pleased to be a *Mecænas* to this, the small Fruits of my Labour, and allow me the Honour to subscribe my self, with the profoundest Veneration and Respect,

May it please your Lordship,

Your Grace's most humbly devoted Servant,

James Douglas.

TO THE

READER.

I *Confess, that to this small Treatise I assume a very high Title; but your Practice with this Instrument, will very soon confirm and demonstrate to you, that of all Instruments (on this Subject) hitherto invented, this is the most* Infallible.

Its not being graduated, gives in the Operation the fewer Measures to be observed, freeing the Practitioner from the tedious Calculation of Angles, forming and infallibly shutting all Sorts of Figures, of what-

whatsoever Angles, either in Planometry, *or* Altemetry, *without the least Error, rendring in all Respects equal Figures, to any given; and that only by parallel Lines hitherto unknown.*

Make Use then of this unparallell'd Instrument, as your Guide in this delectable Study, that you may reap the Profit, and I the Honour, of being useful to you on this Occasion.

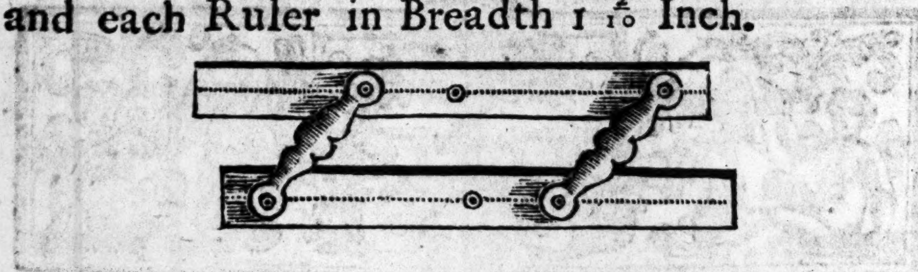
THE

THE SURVEYOR's Utmoſt Deſire, &c.

AS introductive to this new Invention, I find it neceſſary to begin with the Deſcription and Uſe of the parallel Ruler.

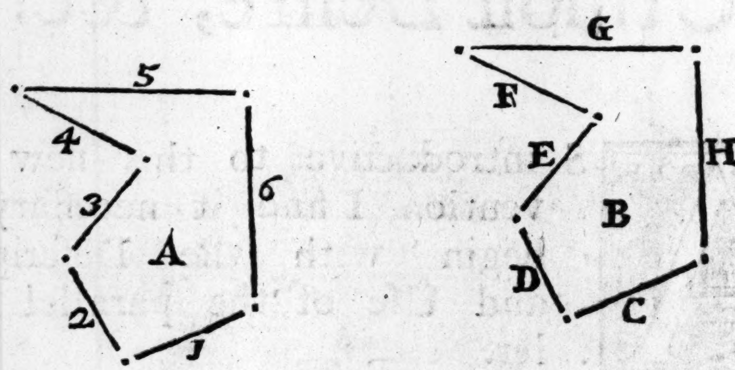
And firſt for its Deſcription, the following View thereof may ſuffice: being in its Structure ſo ſimple, and over all ſo generally well known, that to enlarge on this Subject, it were ſuperfluous; I ſhall therefore proceed to the Uſe thereof: Leaving it to your Option, to have them either of Brazil, Ebony, or Boxwood.

A View of the parallel Rulers, in Length $17\frac{1}{2}$ or 18 Inches, opening to 8 Inches wide, and each Ruler in Breadth $1\frac{2}{10}$ Inch.



Prop. I.

How by the parallel Rulers to copy any given Plane Figure: Example. Let it be required to copy, or tranſcribe the Plane Figure A; *whoſe Sides are marked* 1, 2, 3, 4, 5, 6.



First apply the Edge of the Parallels, to the Side marked 1: Then opening the Rulers to any Diſtance parallel to that Side, draw the Line C, to which give the Length of the 1 Side.

Again, apply the Edge of the Parallels to Side 2; and ſo opening the Rulers, joyn D to C, giving to D the Length of the Side 2.

In

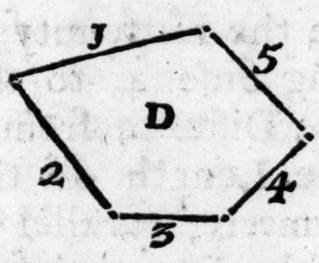
In like manner apply the Edge to Side 3, from thence opening the Rulers, joyn E to D; giving to E the Length of the Side 3; and with your Edge, as aforesaid, at Side 4, opening the Rulers, joyn F to E: make F (in Length) equal to Side 4.

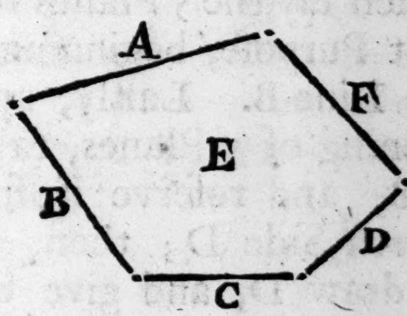
Lastly, Applying your Edge to Side 5; joyn G and F, making G just so long as Side 5; then draw the Line H, and it will be equal to Side *6*; and you will have Fig. B, equal in all Respects to the given Fig. A, which was required.

PROP. II.

To augment or diminish a Plane Figure according to any Proportion given: Example: Let the following Fig. D *be given to be augmented, in proportion as* 5 *to* 8.

TO facilitate this Operation, you must have a Sector whereupon to measure the Lines as followeth:





First apply the Side marked 1, to the opening of 5—5 upon the Line of Planes, and so keep-

keeping the Sector, take the parallel Diſtance from 8 to 8 Planes, which reſerve for the Length of the firſt Side A, of the augmented Plane E: And now applying the Edge of your Parallels to Side 1, open them to any proper Diſtance from the given Plane, and draw the Line A, repreſenting the firſt Side, to which give the Length of 8 Planes aforeſaid.

Again: applying the Side 2, to the opening of 5—5 Planes, take the parallel Diſtance from 8 to 8 Planes, which reſerve for the Length of the ſecond Side B.

Lay the Edge of your parallel Rulers to Side 2, and from thence open them, and draw B parallel to 2, joining at the Extremity of A; to which give the Length of 5 Planes, reſerved as aforeſaid.

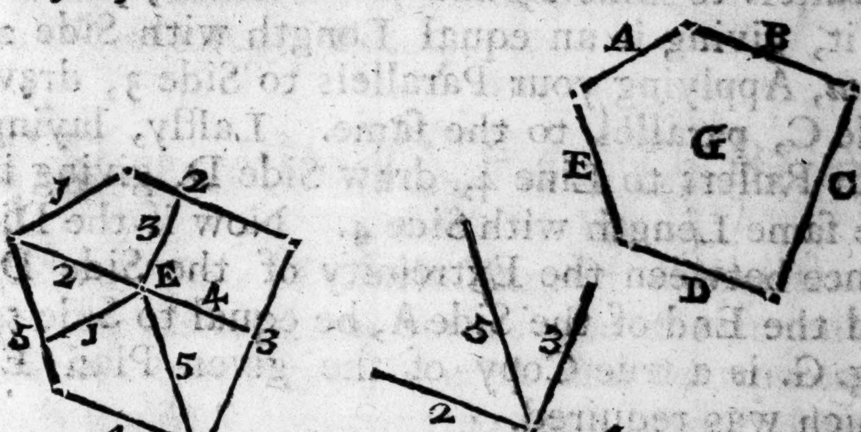
Item. Applying the Side 3, to the opening of 5 Planes, take the parallel Diſtance from 8 to 8, which reſerve betwixt the Points of your Compaſſes for the Length of the third Side C. Then, as aforeſaid, applying the Edge of your Rulers to Side 3, draw C, its Parallel, on which lay the 5 Planes reſerved as aforeſaid for that Purpoſe, beginning from the Extremity of the Line B. Laſtly, applying Side 4, to the opening of 5 Planes, take the Diſtance from 8 to 8, and reſerve it for the Length of the fourth Side D; then, as formerly, parallel to 4, draw D, and give to it the Length of 5 Planes, reſerved as aforeſaid. Take now the Diſtance in your Compaſſes betwixt the two Extremities of the Lines A and D, the which ap-

applyed betwixt 8 and 8 Planes, take the Diſtance from 5 to 5; which, if equal to Side 5 of Figure D, the Plan E is in proportion to Plan D, as 8 to 5. In like manner, if you would diminiſh any Figure; as ſuppoſe the Plane Figure E were given to be diminiſhed in proportion as 8 to 5.

First apply the Side A, to the opening of 8—8 Planes, then drawing the Line 1 parallel to A, make it in Length 5 Planes; and thus proceed till you have cloſed the Figure.

PROP. III.

To copy any Plan from a given Point, taken either within or without its Superficies, Fig. E, F, and G.



EXample. Let it be required to make a true Copy of the Plan, 1, 2, 3, 4, 5. Make Choice then of any Part within its Superficies, where

where make a Point, as at E; then applying the Edge of your Parallels to Side 1, open them to E; and ſo draw Line 1, within the Superficies, parallel to Side 1.

Again, from Side 2, opening to E, draw Line 2, parallel to Side 2; from Side 3, open to E, and draw Line 3; from Side 4, open to E, and draw Line 4; laſtly, from Side 5, open to E, and ſo draw Line 5: And thus have you five Lines central at E, each parallel to their reſpective Sides; whereby to make the true Copy required: Take firſt the Length of the Side 1, in your Compaſs, then apply the Edge of your Parallels to Line 1, and ſo opening your Rulers to any convenient Diſtance, as to A, draw there a ſtreight Line, and make its Length equal to Side 1. Then apply your Parallels to Line 2, and draw Side B, parallel to it, giving it an equal Length with Side 2. *Item*, Applying your Parallels to Side 3, draw Side C, parallel to the ſame. Laſtly, laying your Rulers to Line 4, draw Side D, giving it the ſame Length with Side 4. Now if the Diſtance between the Extremity of the Side D, and the End of the Side A, be equal to Side 5, Fig. G. is a true Copy of the given Plan E, which was required.

To prepare the Point taken without the Superficies, as ſuppoſe at F, firſt lay the Edge of your Parallels to Side 1, and then opening them to the given Point F, draw Line 1; again lay your Edge to Side 2, and opening them to the Point F, draw Line 2. Do the ſame from Side

Side 3, 4, and 5: ſo ſhall the given Point F, be prepared for rendring the true Copy of the given Plan. So if by their Lines, either central within, or without the given Figure, you would deſcribe a true Copy of the ſaid Figure, do as followeth, ſuppoſing thoſe from without the Plan central in the Point F.

Apply the Edge of your Parallels to any of thoſe Lines, as ſuppoſe to that marked 4, and from thence open your Rulers to any Diſtance, as to D; where draw a Line parallel to that you laſt mov'd from, giving to it the ſame Length with the fourth Side in the given Figure.

In like manner from F 3, move your Edge parallel, and draw the Side C, equal in Length to the Side 3; the like of all the reſt till you have ſhut the Figure G; which ſhall be equal in all Reſpects to Fig. E.

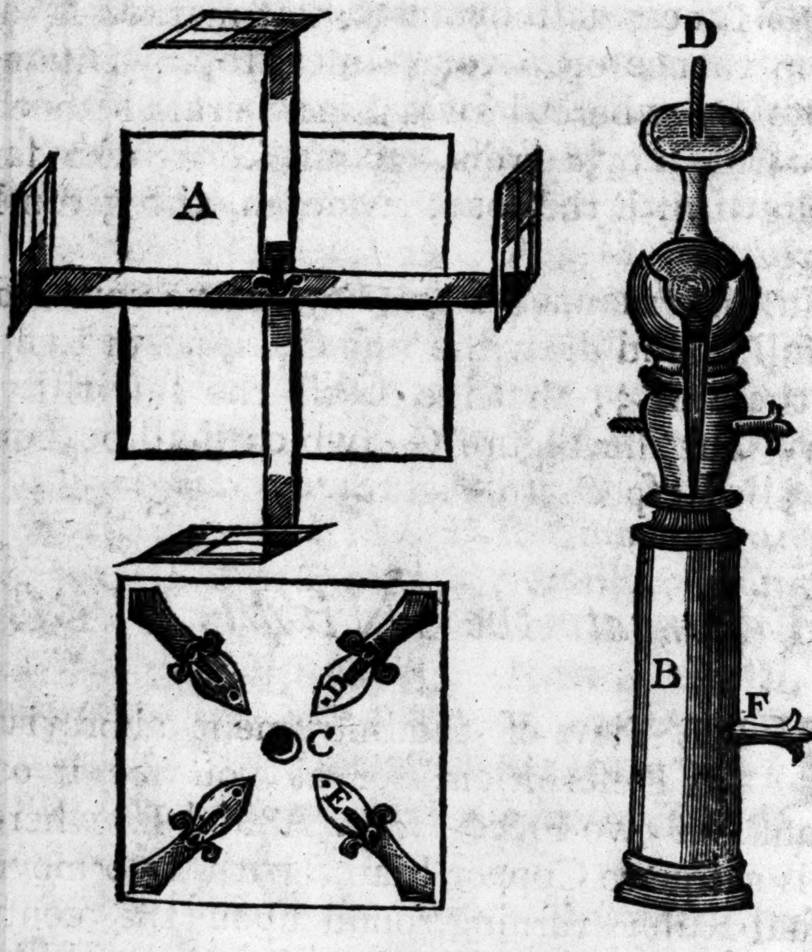
To delineate the INFALLIBLE.

THE View of the Inſtrument abbreviates its Deſcription: for as you ſee it only conſiſts of two Pieces, *viz.* A and B, whereof A is a ſquare Copper Plate, with two moving viſual Rulers turning round upon the central Screw Nail D, paſſing through the middle of the Plate A.

Figure C, repreſents the backſide of the Plate A, furniſhed at each Corner with a thin Piece of Braſs marked D; which may be taken

off and on at Pleaſure, each being pierced at the Point E, to receive the black Points, or headleſs Pins, which are ſoldred faſt to the Plate, the four Screws are to make their Plates hold faſt the paper, when properly folded at the Corners of the Inſtrument.

THE INFALLIBLE IN ALL ITS PARTS.



To fit the Inſtrument for Uſe.

FIRST cover the Plate A, with a double Sheet of clean Paper, neatly folded back-ward

wards along each Side of the Square, then make all the Corners faſt with the aforeſaid Plates of Braſs.

Then B, your Ball and Socket, is to be joined, by putting D, the Screw Nail thereof, through the central Hole of the Plate A, gently piercing the Paper; which done, apply the two viſual Rulers, and ſcrew them faſt with the middle Screw Nail; ſo that the ſaid Rulers move eaſily about the Center: and thus is your *Infallible* prepared for Uſe. After putting in the Button into the Socket, and ſcrewing it faſt with the Nail F, followeth:

The Uſe of the Inſtrument.

PROP. I.

How to take the Plan of any Town, or of any other plain Superfice.

FOR Example, Fig. 1. ſhall be the Situation of a Place, whoſe Plan is required.

Firſt, At all the Angles place Pickets with white Papers, or any thing elſe more remarkable at the Tops thereof.

Then with your Inſtrument at the Angle A, direct the viſual Rulers, the one to B, the other upon G. So that through the Sights you exactly diſcern thoſe two Pickets, and keeping the two Rulers exactly in that Situation, draw with your Lead Pencil a Line along the Edge of each, one of which mark 1, the other 2, ſignifying the firſt and ſecond

Side of the given Figure; and write along 1, 100: and along the Line 2, write 60 equal Parts; ſuppoſing the Sides to be meaſured with the Decimal Chain.

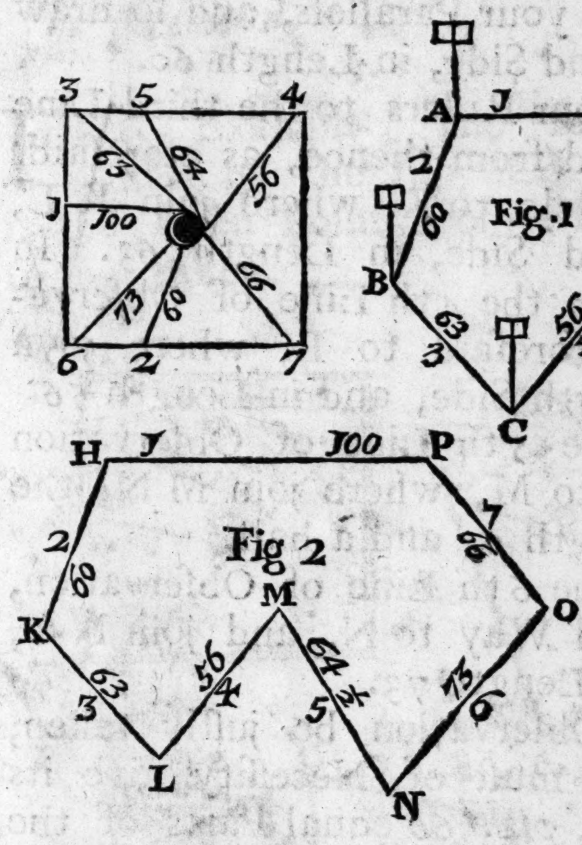
Then remove your Station to B, where direct your viſual Rulers, the one to A, the other upon the Picket C; and along the Edge directed to C, or third Side, draw a Line with your Pencil, and mark it firſt with 3, ſignifying the 3d Side, then write 63, denoting its Length as found by the Chain; which done, remove to the Picket C, where directing one of your Rulers to B, with the other look ſtreight upon D; and along the Edge directed on D, draw a black Line as aforeſaid, which mark with 4, the Number of its Side, and then write 56 equal Parts for its Length by the Chain.

Then remove to D, where direct your Rulers, the one upon C, the other upon E; and along the Edges tending to E, as aforeſaid, draw a black Line, which mark with 5, for Sides, and with 64 and a half equal Parts, for its Length found by the Chain.

Again: Removing to E, direct your Rulers, the one upon D, the other to F; ſo as formerly, draw a black Line, marking it firſt with the Figure 6, ſignifying the ſixth Side; then with 73 equal Parts, its Length by the Chain.

Laſtly, Remove your Station to F, and direct your Rulers, the one to E, the other to G: Now along the Edge directed to G, draw

a black Line, whereupon firſt write 7, for the Number of Sides, then *66*, for its Length, which done, you have all upon your Inſtrument neceſſary to project the exact Figure of the given Plane as followeth:



Diſmount your Inſtrument, and take off the Paper of Obſervations, and unfold it to its full Extent, for Place enough to transfer the Lines of Obſervation in forming the Plan of the obſerved Figure.

Then take your parallel Rulers, whereof apply the firſt Edge to the Line marked 1—100: repreſented in Fig. 1. by the Line AG; and from thence moving parallel to

 any

any convenient Place in the ſame Paper, as to H, where draw the firſt Line of the Plan, or Fig. 2. giving to its Length 100 equal Parts, from any Scale, and mark'd H P.

In like manner, from the ſecond Line of Obſervation move your Parallels, and ſo draw H K, for the ſecond Side, in Length 60.

Then apply your Rulers to the third Line of Obſervation, and from thence, as aforeſaid, move your Parallels to K, where join K L, forming the third Side, in Length 63. In like manner, from the 4th Line of Obſervation, move as aforeſaid to L, where joyn L M, for the fourth Side, and in Length 56.

Next, from the 5th Line of Obſervation move as before to M, where join M N, the fifth Side, in Length 64 and a half.

Laſtly, From the 6th Line of Obſervation, move in a parallel Way to N, and join N O, the 6th Side, in Length 73.

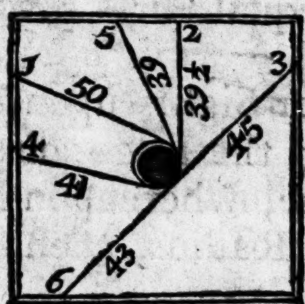
Thus if the Obſervation be juſtly taken, the laſt Side O P, muſt of Neceſſity give its own due Length, *viz.* 66 equal Parts of the Scale, whereof the firſt Side H P, had 100; and ſo infallibly cloſes the Figure, making Fig. 2. in all its Sides and Angles equal to Fig. 1. which was required.

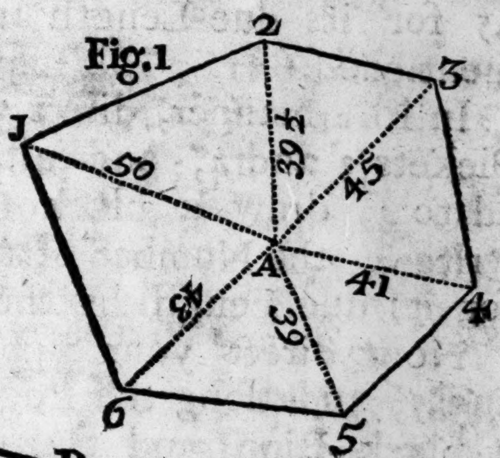
PROP.

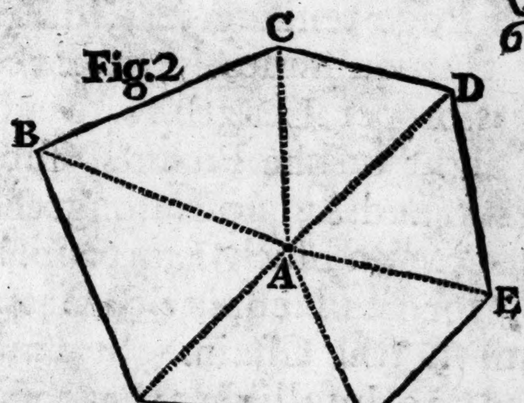
PROP. II.

To take the Plot of any Field from any Point therein, from whence all the Angles may be perceived.

FOR Example, the following Fig. 1. shall be the ſuppoſed Field, wherein A is the given Point from whence the Plot is to be taken.







Having placed your Inſtrument at A, di-

rect one of the visual Rulers to 1, the other to 2; and so along those two Edges, draw with your Pencil a black Line, marking the first with 1, signifying the first Radius, whereupon write 50, for its Length in equal Parts measured by the Chain.

Then mark the second Line with 2, for the second Radius, and $39\frac{1}{2}$ its Length in equal Parts.

Then direct your Sight as aforesaid upon the Angles 2 and 3; so along the Edge directed upon the Point 3, draw a black Line, marking it with 3, for the third Radius, and 45 for its due Length in equal Parts beforementioned.

In like manner, direct your Sights upon the Pickets 3 and 4, and along the Edge directed to 4, draw a black Line, whereupon first write 4, the Number of the Radius, then 41, for its due Length by the Chain.

Now direct your Sight upon the Points 4 and 5, and along the Edge tending to 5, draw a black Line, and thereon write 5, for the Radius, and 39 for its proper Length.

Then direct your Sight on the Pickets, supposed at the Angles 5 and 6; and along the Edge directed to 6, draw a black Line, upon which first write 6, for its Radius, then 43 for its proper Length by the Chain.

So have you upon the Infallible, the compleat Number of Lines wherewith to project the required Plan as followeth:

Having,

Having, as before taught, taken down your Inſtrument, and unfolded the obſervatory Paper, make Choice of any convenient Point thereon, as at A: Fig. 2. to repreſent your ſtationary Point, to which, from the firſt Line of Obſervation, move your Parallels, and draw A B, parallel to the Side Line, repreſenting A 1, in Fig. 1. to which give 50 equal Parts in Length from your Scale.

In like manner from the ſecond Line of Obſervation, move with your Parallels, and draw the Line A C, equal to A 2 in Length, and parallel to the ſame.

Again, draw A D, parallel to the third Line of Obſervation, and in Length 45 equal Parts, repreſenting the Radius A 3; then applying your parallel Rulers to the *4th* Line of Obſervation as aforeſaid, draw A E parallel thereto; upon which ſet off 41 equal Parts, repreſenting A 4.

Now from the *5th* Line of Obſervation, move with your Parallels as aforeſaid to A, where draw A F: giving to its Length 39 equal Parts, equal to A 5.

Laſtly, in manner aforeſaid, draw A G, parallel to the *6th* Line of Obſervation, to whoſe Length give 43 equal Parts, repreſenting A *6*; which done, the laſt Diſtance, *viz.* G B, will fall out equal to *6*: 1: Fig. 1. and is the Proof of the Work.

And thus have you the ſix Radius's, each of a due Length, about which deſcribing ſtreight

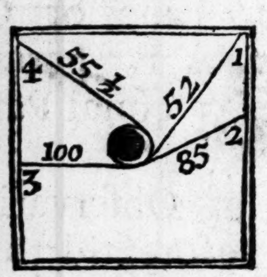
Lines,

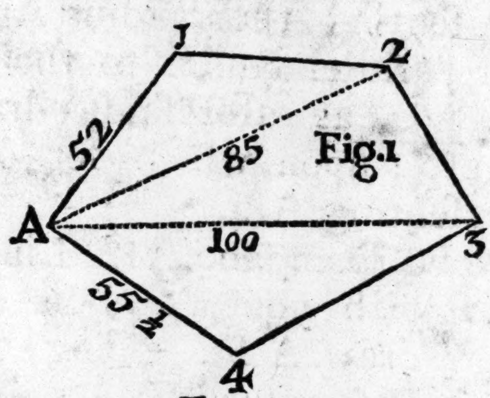
Lines, as AB, CD, DE, EF, FG, and GB, will produce Fig. 2. for a true Plan or Plot of the given Figure, which was required.

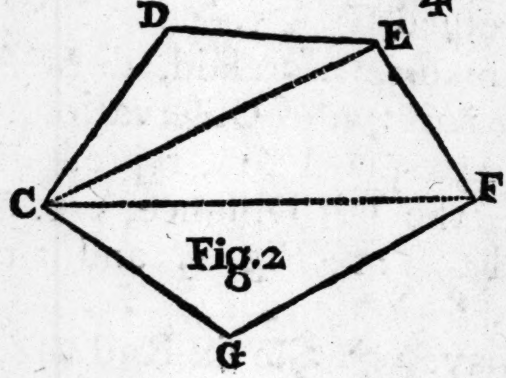
PROP. III.

To take the Plot of a Field from any angulate Point thereof, from whence all the other Angles may be diſcovered.

FOR Example, the following Fig 1. ſhall be the ſuppoſed Field to be ſurvey'd, where A is the angular and ſtationary Point, from whence the Obſervation is to be made.







Place

Place the Inſtrument at the aſſigned Point A, where directing your viſual Rulers, the one to 1, the other to 2, along each of the Edges draw a black Line, and mark the firſt 1 for Radius, as alſo with 52 equal Parts for its Length, meaſured with the Chain; the other likewiſe with 2 for Radius, and 85 equal Parts for its Length.

Then proceed, directing your Sights as aforeſaid to 2 and 3; and along the Edge directed to 3, draw a black Line, whereupon write 3 for the third Radius, and 100 equal Parts for its Length.

Laſtly, direct your Sights upon the Angles 3 and 4; ſo along the Edge, pointing to 4, draw as aforeſaid a black Line, whereupon write 4 for the Radius, and 55½ equal Parts, its Length meaſured as aforeſaid.

And ſo have you all the Lines upon your Inſtrument, requiſite to project the Plan of the given Field as followeth.

Having as formerly diſmounted your Inſtrument, and unfolded the obſervatory Paper, chooſe any Part thereon, as at C, Fig. 2. to repreſent your ſtationary Point.

Then apply your parallel Rulers to the firſt Line of the Obſervation, and from thence opening them to C, draw CD, and give to its Length 52 equal Parts, which repreſents the Side A 1, in the given Figure.

Secondly, Applying your Parallels to the ſecond Line of the Obſervation, open them

to

to C, and draw C E; giving to its Length 85 equal Parts, to repreſent A 2.

Thirdly, From the third Line of Obſervation, open your Parallels to C, where draw CF; to which give 100 equal Parts in Length, to repreſent Radius A 3.

And laſtly, From the fourth Line of Obſervation move your Rulers to C, where draw C G, and make it 55 equal Parts in Length, repreſenting Side A 4.

Now round the Extremities of all their Diagonals, draw ſtreight Lines; and you ſhall have made Fig. 2. equal in all Reſpects to Fig. 1. which was required.

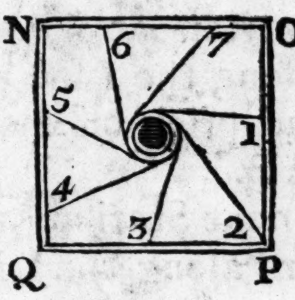
Prop. IV.

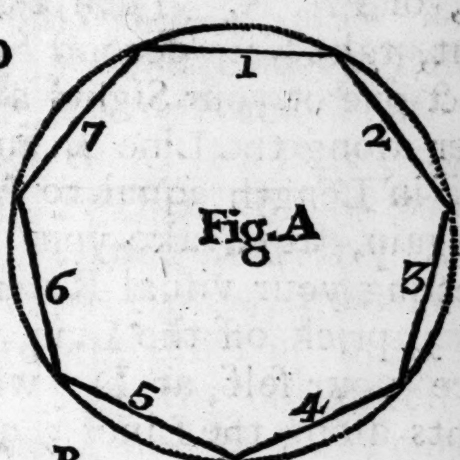
To lay down, or trace any Poligon, or given Figure upon the Ground.

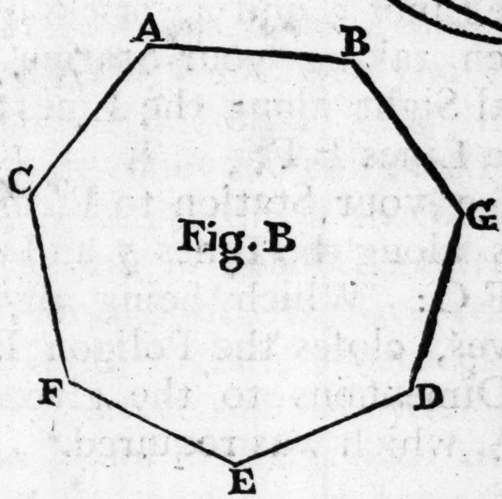
FOR Example: Let it be required to lay down an Heptagon.

Upon a Sheet of clean Paper firſt deſcribe an Heptagon, as Fig. A; next to which make a Square upon the ſame Paper, juſt to cover your Inſtrument, as marked N O P Q; round the Center whereof firſt deſcribe a little Circle, whoſe Diameter muſt be a little longer than the Breadth of one of your viſual Rulers; to the end that in Field Operation, the Lines deſcribed upon the ſaid Square, may not be hid by the Breadth of the viſual Rulers; and within

within this Circle, as you ſee in the Figure, a Hole is to be cut out juſt in the Center, no bigger than to receive the Screw Pin of the Ball and Socket, which done, from all the Sides of Fig. A. move your Parallels to touch the biggeſt Circle within the Square, and draw the correſpondent Lines parallel Lines, marking them 1 2 3 4, *&c.* to 7, the Number of Sides in the given Figure.







Thus all the obſervatory Lines being drawn upon the Square, cover your Inſtrument there-with,

with, and fit it for Uſe: Which done, upon the Ground appointed take A for your firſt Station, and there planting your Inſtrument, direct the two viſual Rulers, the one along the Line 1, the other along the Line 7; and ſo according to the Direction of thoſe two viſual Rulers, draw ſtreight Lines from A, to B and C, placing a Picket at each Point; the Diſtance of each Picket from one another equal to the Length of the Side of the given Poligon, or Fig. A. Then removing your Inſtrument, take your ſecond Station at B, where direct one of your Sights along the Line 1, the other along the Line 2, and prick off the Line BG, in Length equal to AB.

Again, at G take your next Station, where directing your viſual Rulers along the Lines 2 and 3, prick off the Line, or Side DG; then place your ſelf at D, where directing your Sights along the Lines 3 and 4, prick off the Side DE: Then taking your Station at E, direct your viſual Sight along the Lines 4 and 5, and draw the Lines EF.

Laſtly, Change your Station to F, and direct your Rulers along the Lines 5 and 6; ſo draw the Side FC: Which being all equal among themſelves, cloſes the Poligon Fig. B. equal in all its Dimenſions to the given Heptagon, or Fig. A. which was required.

PROP. V.

How at two Stations to measure inaccessible Distances. Fig. 1.

FOR Example: Let it be required to measure the Distances of the Trees marked 1, 2, 3, 4, all inaccessible because of the River.

First, along the River-Side draw a right Line, as H H, for your stationary Distance, giving to its Length any Number of equal Parts you please, as here 100 Paces, placing a Picket at each End; then choose your first Station at the right Hand, and from thence direct your two visual Rulers, the one upon the Tree marked 1; the other upon the Picket on the left Hand: So with your Pencil draw the two black Lines 1 and H, signifying both the directory Line to the Tree, and the stationary Horizontal Line H — H: Further, keeping one Ruler exactly upon the Left Hand Picket, direct the other to the second Tree, and so upon the third and fourth.

Which done, remove your Station to the left Hand Picket, where planting your Instrument, direct one of your Sights upon your right Hand Picket, with the other upon the Tree 1; and so as formerly draw the two Lines 1 and H upon the Instrument; and in this manner continuing still one Sight directed upon the right Hand Picket, and with the other

other interſecting all the viſual Lines of the firſt Station, at the ſeveral Objects marked 1, 2, 3, 4, every Time carefully drawing your directory Lines upon your Inſtrument, you will have got all the Lines neceſſary for deliniating the Plan of the required Diſtances as followeth.

Take down your Inſtrument, and unfold its Paper Cover, and thereupon with your parallel Rulers carry off the Horizontal Line H, to A B; to whoſe Length give 100 equal Parts, repreſenting H—H, the ſtationary Diſtance upon the Field.

At the Point A, join with your parallel Rulers (as aforeſaid) all the obſervatory Lines of the right Hand, and in like manner all thoſe of the left join at B; all which being ſufficiently produced, will interſect each other at the Points 1, 2, 3, and 4, denoting the true Diſtances of thoſe Trees from each other, as alſo from your ſtationary Diſtance.

The Proof of the Work is thus:

The Diſtance H 1, or the firſt Tree from the firſt Station, meaſured upon the Scale, (whereof H—H is 100) will give $74\frac{1}{2}$; and A 1 its Parallel meaſured upon the Scale (whereof A B is 100) will give alſo $74\frac{1}{2}$: for as A 1 is to A B: ſo is H 1 to H H.

Again: Meaſure the Diſtance H 2, upon its proper Scale, and you'll find it contains $96\frac{1}{2}$; then meaſure A 2 its Parallel, upon Scale A B, and it will be the ſame; and thus you'll find the

the third visual Line to be 82; and the fourth $119\frac{1}{2}$; which are the true Distances of those four Objects from the first Station; and thus may the Map of any Lordship, or small Country, be taken with sufficient Exactness.

Prop. VI.

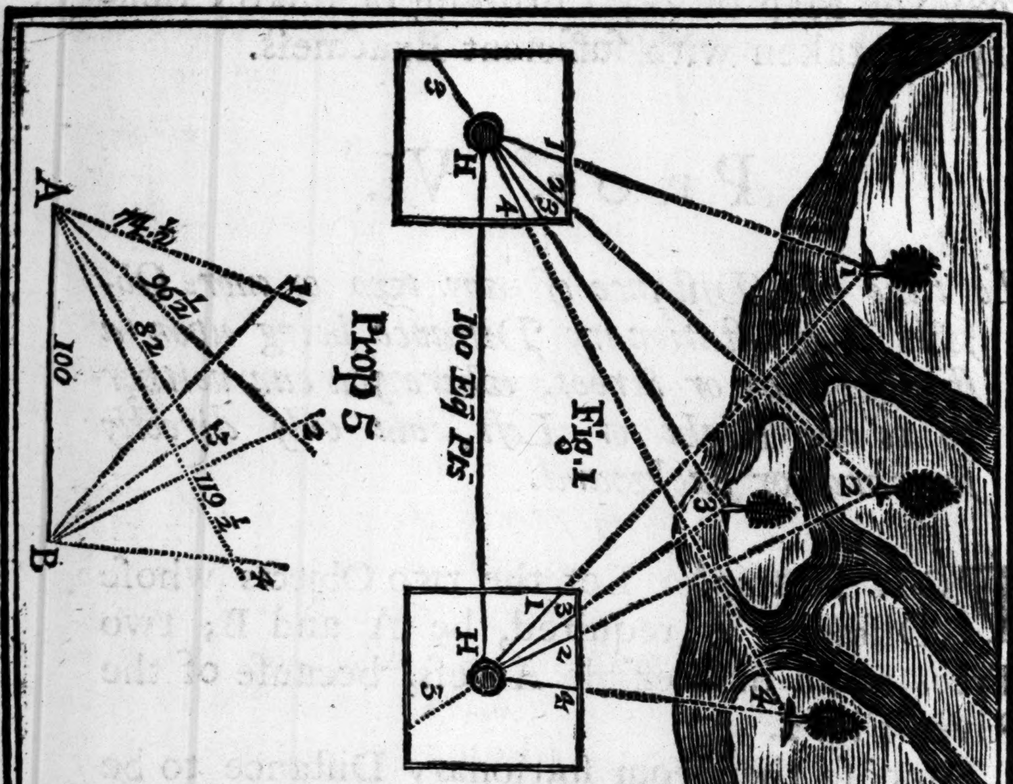
To take the Distance of any two or more Objects, your stationary Distance being upon a streight Way or Street, where you can neither go to the Right or Left, but only directly forward or backward.

FOR Example: Let the two Objects whose Distance is required, be A and B; two Trees of an inaccessible Access, because of the River Fig. 2.

Suppose then your stationary Distance to be the Line H——H; of 100 equal parts in Length, and from the End thereof nearest the River Side make your first Observation, by directing your Sight through your visual Rulers, to the two Trees A and B; and so draw the two black Lines along the Edge of the Rulers, marking them 1 and 2.

Then remove your Station backwards to the other End of the Horizontal Line, where plant your Instrument, and first through one of the visual Rulers, observe the Picket at the first Station; and accordingly upon the

Instrument draw the black Line H—H, for your Horizon or Basis.



Then

Then through the two viſual Rulers, as before taught, obſerve again the two given Objects A and B, and mark thoſe obſervatory Lines ſomewhat diſtinct from the others, as with a red Pencil, *&c.* then write upon the firſt 1, and upon the ſecond 2.

Laſtly, To obtain the required Diſtances, take down your Inſtrument, and unfold its Cover, or obſervatory Paper; then with your Rulers draw a Parallel as aforeſaid to the Horizontal Line H—H: as DE: and thereupon ſet off 100 equal Parts of a leſſer Scale.

From the Point E, draw the Lines EF, and EG: parallel to the two Lines of the firſt Obſervation marked 1 and 2. In like manner from D, draw DF, and DG, parallel to the two obſervatory Lines of the laſt Station, whereby the two viſual Lines of the firſt Operation ſhall be interſected at the Points F and G, giving FG for the true Diſtance betwixt the two given Objects; which, meaſured upon the Baſe Line DE, will give 70 equal Parts; the Side DF, 150 $\frac{1}{2}$; and the Side DG, 167: the Proof whereof is, meaſure AH, upon HH, its proper Baſis, and you'll find its Length 150$\frac{1}{2}$ equal Parts: HB: 167: and AB: 70. which was required.

PROP. VII.

To measure all Sorts of accessible and inaccessible Heights.

AS for Example: Let it be required to take the Height of the following Tower AB; first upon its accessible Side, and then from its inaccessible Side, by reason of the River.

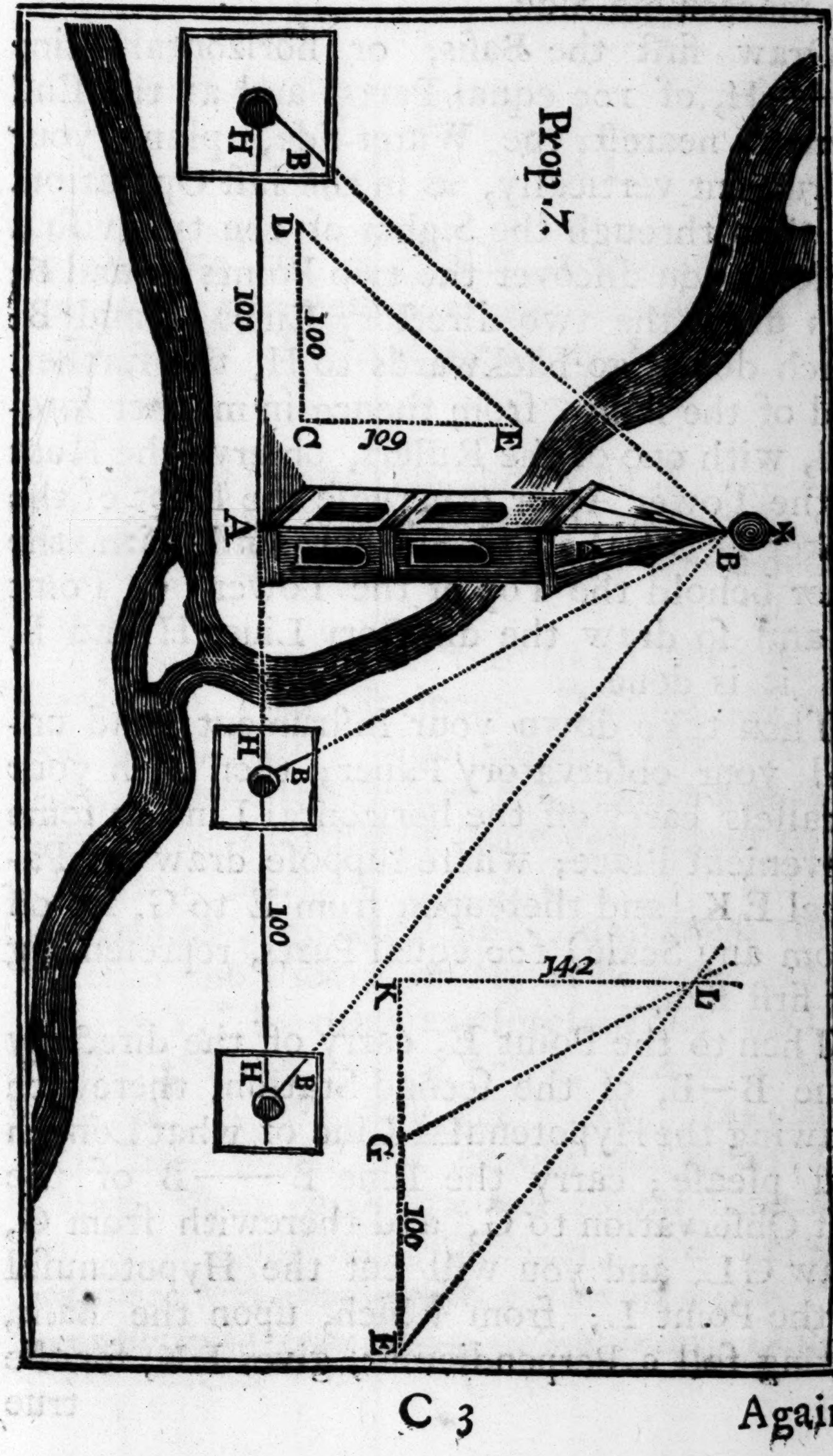
First upon the accessible Side, draw the Basis, or Horizontal Line AH; in Length 100 equal Parts; then plant your Instrument, turning the Ball within the Socket, so that one of the Sides of the Square be brought perpendicular to the Horizon.

Then direct the two visual Rulers, the one to discover the Root of the Tower at A, the other the Top at B; and then draw the two directory Lines B and H, and it is done.

Next, to transfer the Figure as formerly, take down your Instrument, and unfold the observatory Paper; then with your parallel Rulers carry off H —— A, the horizontal Line, to a convenient Place, as to D; where draw its Parallel DC, of 100 equal Parts: In the same manner carry off the Line B—H to the Point D, and draw the Hypotenusal as far out as you please.

Lastly, Upon the Point C raise a Perpendicular, and it will cut the Hypotenusal in E, giving the Length CE, for the true Height re-

required; which, meaſured upon the Baſis CD, will contain 109 of thoſe equal Parts.



 Again;

Again: To meaſure the ſame Height from the inacceſſible Side.

Draw firſt the Baſis, or horizontal Line H——H, of 100 equal Parts, and at the End thereof neareſt the Water-ſide, plant your Inſtrument vertically, as in the laſt Operation. So that through the Sights of the two viſual Rulers, you diſcover the two Points A and B; then draw the two directory Lines H and B; which done, go backwards to H, the furtheſt End of the Baſis; from thence in manner foreſaid, with one of the Rulers, obſerve the Root of the Tower A, or otherwiſe the Root of the Picket H, of the firſt Station, and with the other behold the Top of the Tower, or Point B, and ſo draw the directory Lines H and B, and it is done.

Then take down your Inſtrument, and unfold your obſervatory Paper; then with your Parallels carry off the horizontal Line to ſome convenient Place; where ſuppoſe draw its Parallel E K, and thereupon from E to G, ſet off (from any Scale) 100 equal Parts, repreſenting the firſt Baſis.

Then to the Point E, carry off the directory Line B—B, of the ſecond Station, therewith drawing the Hypotenuſal Line of what Length you pleaſe; carry the Line B——B of the firſt Obſervation to G, and therewith from G, draw G L, and you will cut the Hypotenuſal in the Point L; from which, upon the Baſis, letting fall a Perpendicular, gives L K, for the true

true Height of the Tower; which, meaſured upon EG, its proper Baſis, give 142 of thoſe equal Parts. For as AB is to HH, ſo KL is to EG.

PROP. VIII.

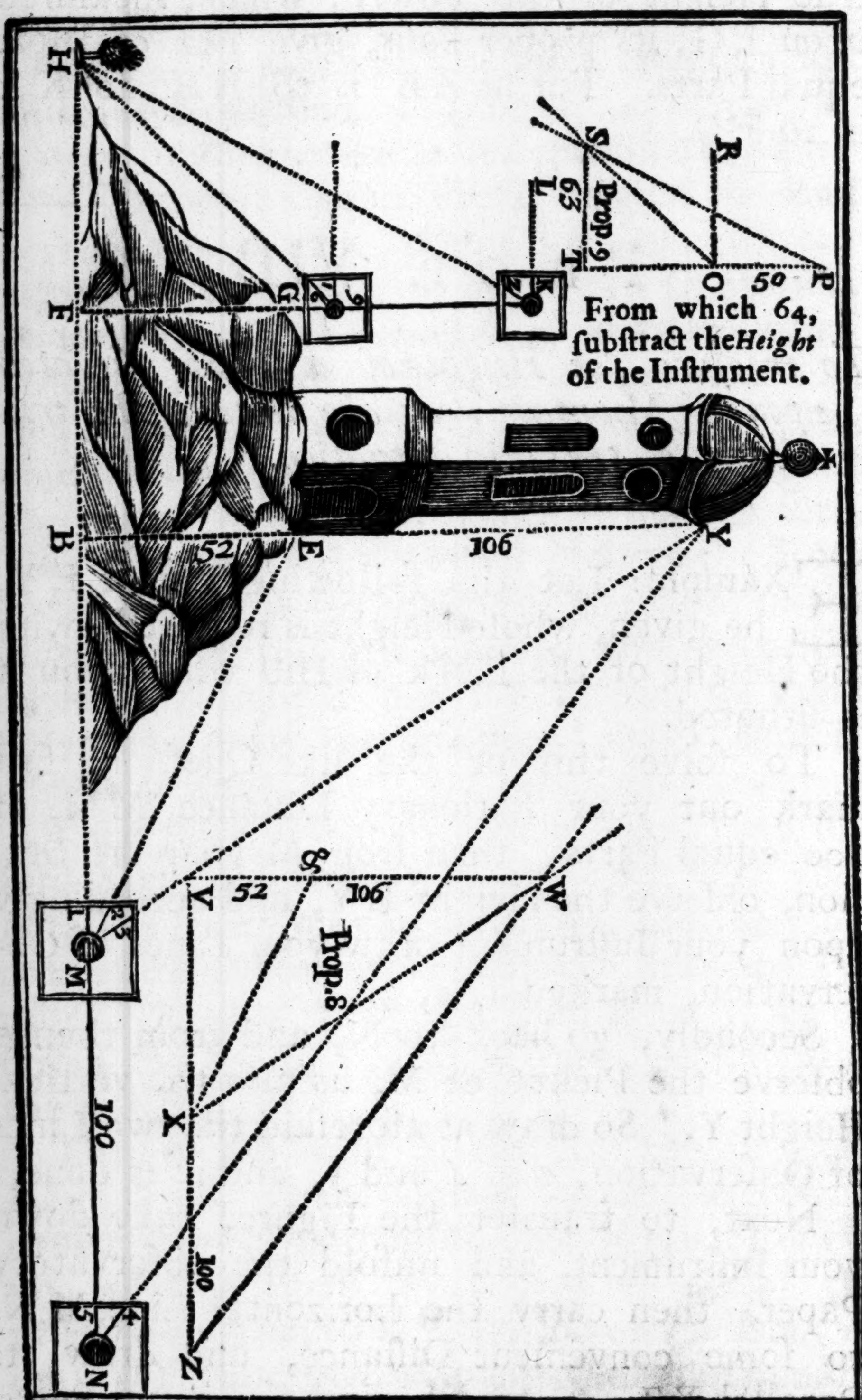
To meaſure the Height of a Tower ſituated upon a Mountain; as alſo to take the true Height of the ſaid Situation.

EXample: Let the following Tower EY, be given, whoſe Height is required; with the Height of the Rock or Hill whereupon it is ſituated.

To ſolve this or the like Queſtion, firſt mark out your ſtationary Diſtance MN, of 100 equal Parts; then from M your firſt Station, obſerve the Height EY, and accordingly upon your Inſtrument draw your Lines of Obſervation, marked 1, 2, 3.

Secondly, go back to N, and from thence obſerve the Picket at M, as alſo the vertical Height Y. So draw as aforeſaid the two Lines of Obſervation, *viz.* 4 and 5, and it is done.

Next, to transfer the Figure, take down your Inſtrument, and unfold the obſervatory Paper, then carry the horizontal Line MN, to ſome convenient Diſtance, and draw its Parallel XZ; to which give 100 equal Parts, repreſenting your Baſis, or ſtationary Diſtance.



Then parallel to the observatory Line 2,
draw

draw X &; and parallel to the obſervatory Line 3, draw X W.

Laſtly, Parallel to Line 5, draw Z W, and let fall the Perpendicular W V, and you ſhall obtain & W, for the true Height of the Tower, with & V, the Height of its Situation.

Laſtly, To determine the Height of the Tower, meaſure E Y upon M N, its proper Baſis, and you ſhall find it contains 106 of thoſe equal Parts; whereof B E hath 52 for the Height of the Situation above the Horizon.

And by the Parallels meaſure W &, upon X Z, its proper Baſis, and it will be found 106 of thoſe equal Parts, for the Height of the Tower; and in like manner & V being meaſured, will be found 52 equal Parts in Length, which is the Height of the Situation.

Prop. IX.

To meaſure the Height of a Mountain only from one Station thereupon; from which the Foot of the Mountain may be diſcovered.

TAKE your Station at G, where plant your Inſtrument, with the Baton ſo ſunk in the Ground, that being behind it you may conveniently perceive the Foot of the Hill H; and ſo draw the obſervatory Lines, *viz.* 1 for the Horizon, and 6 for the Slope

of

of the Hill; which done, raiſe the Inſtrument at its full Height, ſuppoſe 50 equal Parts, and ſo direct one of your viſual Rulers horizontally, and draw the Line K——L, then through the other Ruler beholding H, draw the Line 7, and it is done.

Laſtly, To form the Figure of the required Height FG, take down your Inſtrument, and with your Parallels transfer the Line 1, to O, and draw O——R, and O——P, make Parallel to the obſervatory Line 9, giving to its Length 50 equal Parts, repreſenting the Height of the Inſtrument from G.

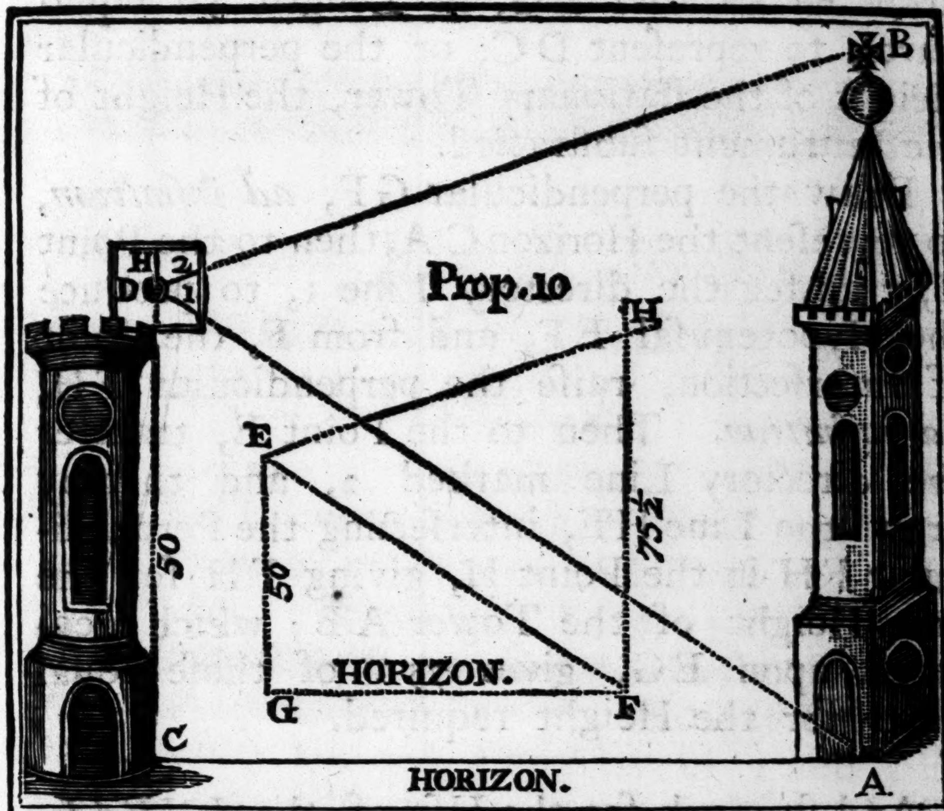
Now from the Point P, draw PS, parallel to the obſervatory Line 7; and from C draw CS, parallel to the obſervatory Line 6. So from S, the Interſection of thoſe two Lines, let fall the perpendicular ST, and it will give TO, for the Height of the Hill; which, meaſured upon OP, gives 64 of thoſe equal Parts, repreſenting FG; and ſo will ST, meaſured upon OP, give $64\frac{1}{2}$, repreſenting HF. For as ST is to OP, ſo is FH to GK, *&c.*

PROP. X.

Being upon one Height, to meaſure the Height of another.

FOR Example: Being upon the Top of the Tower at D, I demand the Height of the Tower AB.

Through



Through the two viſual Rulers from your Station at D, obſerve the Points A and B of the oppoſite Tower, drawing along the Edges the repreſentative Lines of Obſervation, as marked upon the Inſtrument 1, 2; mark likewiſe the perpendicular Line H, and meaſure the perpendicular Height DC, which ſuppoſe to be 50 equal Parts; and thereby have you all the Lines requiſite for ſolving this Propoſition.

Laſtly, To transfer the Figure, and find the required Height; take down your Inſtrument, and unfold the obſervatory Paper; then uſe your Parallels, and put the Line H to E, where

draw

draw its Parallel EG, in Length 50 equal Parts, to represent DC, or the perpendicular Height of the stationary Tower, the Height of the Instrument substracted.

Draw the perpendicular GF, *ad infinitum*, to represent the Horizon CA, then to the Point E, transfer the directory Line 1, to produce the Hypotenusal EF, and from F, the Point of Intersection, raise the perpendicular FH, *ad infinitum*. Then to the Point E, transfer the directory Line marked 2, and thereby draw the Line HE, intersecting the Perpendicular FH in the Point H, giving FH for the true Height of the Tower AB; which measured upon EG, gives 75½ of those equal Parts for the Height required.

And so much for the Use of the *INFALLIBLE*, leaving the further Improvement to the more ingenious.

The

The common Way of Surveying with Gunter*'s Chain, rectify'd by the INFALLIBLE.*

FIRST know, that in *England* three Barley-corns makes one Inch, twelve Inches one Foot, 3 Feet one Yard, $5\frac{1}{2}$ Yards, or $16\frac{1}{2}$ Feet makes one Rood, Perch, Lug, or Pole, 40 Perches in Length, and 4 in Breadth, or which is all one, one Acre of Land containeth 160 Square Roods, or multiply 40 by 4, and it will produce 160. *Item*, One Furlong is 40 Poles or Perches in Length, and eight Furlongs goes to one Mile.

Now Mr. *Gunter*'s Chain, at present the only in Use, is in Length 4 Poles, or 66 Foot, for multiply $16\frac{1}{2}$ by 4, and it will produce 66, which divided into 100 equal Links, forms the Chain, and 10 square Chains makes one Acre. Arithmetically

Thus —— 100
100

$$\left.\begin{array}{r}10000\\10\end{array}\right\}\ 100000\ \text{facit.}$$

PROP.

PROP. I.

How by the foresaid Chain alone, to survey any Piece of Land, be it ever so irregular, and to calculate the Content thereof in Acres, Roods and Perches.

HERE the Field to be surveyed, is ABLMNOPQ, with its Off-sets C D E F, *&c.* along its three Sides to M. Fig. 1.

First from A, running along your Chain to B, you shall find the Side A B, of 600, or 6 Chains Length; but because from A to C there is a Bending in the Hedge, which measured with the Chain gives 50 Links, in the Field Book annexed to the Fig. Under Column marked Distance I set down O O O, my first Station being only the Beginning of the Side AB, but write 50 under the Column of Off-sets, for the perpendicular Length A C.

Then a Chain further from A to b, I find the Angle D; to which measuring from b, I find 100, or one Chain; therefore in the Book under Distances I set down 100, and under Off-sets also 100, for the Length b D.

Then from b to C, the Bend E appears, to which measuring from C, finds it 70 Links in Length; therefore under Column Distance write 127, and under Off-sets 70. And so on observing the same Order till you come to B, then

then will you have 7 Off-ſets, amounting in all to 574, and the Diſtances to 600, equal to Side A B.

And becauſe the Off-ſets are ſeven in Number, divide their Sum 574 by 7, and the Quotient will be 82; by which 600, or A B, the Baſis of all the ſeveral Trapezia, and you'll have 49200, not amounting to one Acre; wherefore to find its Value in Roods, multiply it by 4, the Number of Roods in one Acre, and it will Produce 196800.

Laſtly, To find the Value of 96800, multiply it by 40, the Number of Perches in one Rood, and you will find 3872000. Therefore the total Content of the Off-ſets upon Side A B, is 00 Acres, 1 Rood, 38 Perches, neglecting the Fraction 72000, as uſeleſs in Practice. And through the Field Book draw now a Line, ſignifying you have done with the firſt Side.

Take now your ſecond Station at B, and from thence meaſuring to L, you will find 555, but at the Point B, I meet with one Off-ſet of 47 Links; therefore in my Field-Book, under Diſtance I write 00, but under Off-ſets I mark down 47. Then going on towards L, I find that at 142 Diſtance from B, the Chain touches the Hedge at Y; therefore under Diſtance I write 142, and under Off-ſets the Word Touch. Then from Y to L, at the Diſtance of 413, I take only the Off-ſet L Z of 154, andplace it as in the Book under the Column marked

marked Off-sets, and 413 under Distances. Lastly, because the Diagonal AL forms the Angle at B, let AL be measured, and you will find it 1055; which set down as in the Book under Diagonal, then draw a Line quite through the Book, and summing up the Distances, you will find 555 for the Length of the Side BL; and so have you done with the second Side.

As for the third Side, first from L, as a Center, take the three diametrical Off-sets, forming the Bow Z & ♈♉ the first of 173, the other of 165, and the third of 130; all which set down in the Field Book, as also 430, the Length of the Side L M.

But on my Way from L toward M, distant from L 79 Links, I find at ♊ one Off-set of 28. Therefore under Distance write 79, and under Off-sets 28, and so going on 39 further to ♌. I perceive one Off-set of 80 in Length, set therefore 39 under the Column Distance, and 80 under Column Off-sets; and in this order measuring all the Way to M, I find the Sum of all the Off-sets amounts to 614, and the Number of their Perpendiculars to be 9. Therefore dividing 614 by 9, gives 68 in the Quotient, and thereby multiplying 351, or ♊ M, the full Basis of all the Perpendiculars of the third Side, you will have for Product 23868; which as aforesaid multiplied by 4 yields 95472, and that again multiplied by 40, gives 38 Perches, throwing away the decimal Fraction

ction 18880; which done, draw a Line quite through the Book, as having finiſhed the third Side; but meaſure the Diagonal B M, and you will find it 825; which ſet down as in the Book, being the Baſis of the Angle at L; alſo inſert the three poligonal Lines Z &: & ♈: and ♈♉, under the Column of Contingents, which finiſhes the third Side.

Now being at M, meaſure the fourth Side M N 364, with its diagonal L N 572; and book each in their proper Place, *viz.* 364 under Sides, and 572 under Diagonals; then at N take the fifth Side N O 293, with the Diagonal B O 825; which alſo ſet down as aforeſaid in their proper Places.

Which brings me to O, where meaſuring the ſixth Side O P 530, with its Diagonal O Q 424, ſetting them down as in the Book, together with the Diagonal N P, Baſis of the Angle at O, and ſo advance to P, where meaſure the 7*th* Side P Q 475. Laſtly, Being at Q, take the Side Q A 409; all which book'd in their proper Columns, gives you the true Plan of the given Figure or Field.

Prop. II.

How by the Help of the Field Book to project the true Figure of the given Field upon Paper. Fig. 1.

UPON a Sheet of clean Paper draw the Line A B, on which from your decimal Scale ſet off 600, repreſenting the Side A B; then from the ſaid Scale take 555, and there-with from B ſweep an Arch, and with the Diagonal A L, from A, interſect the ſaid Arch at L; draw B L to repreſent the ſecond Side, upon which at due Diſtance raiſe all the Orthogonal Lines, to form the ſeveral Off-ſets belonging to the firſt and ſecond Side.

Then with 430 of the ſaid Scale ſweep an Arch; which with 825 from B interſect at M, draw L M, repreſenting the third Side of the Field.

Now at M, with 364, ſweep an Arch; which with 572 from L, interſect at N, then draw M N for the fourth Side.

Next from N with 293, deſcribe an Arch, and with 825 from B, interſect the ſame at O.

Again, From the Point O, with 530, ſweep an Arch, which from N, with the Diagonal NP 735, interſect at P draw the Line O P,

O P, to represent the sixth Side, as N O did the fifth.

Lastly, Being at P, with 475, taken from your decimal Scale, sweep an Arch; which from O, with the Distance of 424, intersect at Q, draw the Line P Q for the seventh Side. Then draw the Line A Q, which measured if it be in Length 409, as in the Book, the Figure is rightly shut; and here you may observe the Angle O P Q is in Quantity 424 Links, and so is the Angle L M N of 572, *&c.* which done, raise all the Orthogonals, or perpendicular Lines, drawing all the other Lines each in their proper Lengths; and thereby are formed all the Off-sets belonging to the second and third Side, rendring at the same Time the true Figure of the observed Field.

PROP. III.

To find the superficial Content of Fig. 1. *in Arches, Roods, and Perches.*

BEfore I begin with the inward Superficies, I shall measure the outward Content of the Bends and Off-sets forming the irregular Ground, upon the Outside of the Line B L, beginning at the Point K, and ending at II.

First, Considering that the Off-sett 47, is the Perpendicular of the Triangle K B Y; whose Basis is K Y 248, multiplied by $23\frac{1}{2}$;

 the

the half Perpendicular gives 5828 for the Superficies, which set down as annexed by the Figure.

Now observing that the irregular bending of the Hedge from Y to Z, will best admit of a triangular Shape, I draw the linked Line Y Z; whereby having as much of the Bending without as within the Line Y Z, I multiply Y L 413, all the Basis by 77, half the Perpendicular L Z, which produces 31801 for the Superficies, which I place and notify by its proper Letter, under the last found Superficies.

In like manner I take all the Superficies of the other four Triangles of the Bend Z & ♈ ♉ ♊ and set them all orderly each with their proper Characters, under the other two last found Superficies, whereof the total Sum amounts to 60782, which not being one Acre, I multiply by 4, the Number of Roods in one Acre, and it produceth 2,43128; from which cut off by a Point the five last Figures, and there will remain towards the right 2 Roods, with a Decimal Fraction of 43128; which again I multiply by 40, the Number of Perches in one Rood, and it yields 17,25120; from which as aforesaid cutting off the five last Figures, remains towards the right 17 Perches, $\frac{25120}{100000}$ being the 100000 Parts of one Perch; and because that in 100000, or in one square Acre, there's five Cyphers from the left Hand, of all these Kinds of Sums, five Figures must still be cut off by a Point, being the

the Fractional Part of the Integer, Figure, or Figures, ſo cut off, from that Sum to the Right.

Thus all the outſide irregular Ground and Off-ſets is duly ſurveyed, as appears by the Order of the Field Book.

For firſt, as to the ſix Off-ſets, or Trapezia of the Side AB, their Sum amounting to 574, divide that by 7, *viz.* one more than the Number of Trapezia, (which obſerve for a Rule in all ſuch Occurrences) and the Quotient gives 82; which multiplied by AB 600, the Length of the common Baſis, gives 49200, and that multiplied by 4, the Number of Roods in one Acre, produces one Rood, and the Decimal Fraction 96800; which multiplied by 40, the Number of Perches in one Rood, produces 00 A. 1 R. 38 P. for the compleat Superfice of all the Off-ſets along the Side A B.

Laſtly, For the Off-ſets upon the Side L M, they only begun at the Point ♊; becauſe from L to ♊ makes only the Baſis of the Triangle L ♉ ♊ of 79 Links, whereof (as in the Field Book) from L M 430, take L ♊ 79, and there remains 351; for the whole Length of the common Baſis, of all the eight Trapezia, from ♊ to M, the Sum of whoſe Perpendiculars (as in the Book) amounts to 614; which for the Reaſon aforeſaid divide by 9, being one more than the Number of Trape-

zia, upon the Line ɪɪ M, gives 68 in the Quotient, and thereby multiplying 351, the common Basis, gives 23868; and that multiplied by 4, the Roods in one Acre, gives 95472; which multiplied by 40, the Perches in one Rood, produceth 38,18880, being only 38 Perches, neglecting the Fraction.

I now proceed to measure the Superfice of the inward Poligon or Plot; for which End the throwing of the whole Field into Triangles, is the most expeditious Method; as followeth.

Draw first the occult Line L Q, then QB and QN, which, with the other Diagonals already drawn, divides the whole Field in six Triangles, whose Perpendiculars and Bases you may by Inspection easily distinguish.

To find the superficial Content of the first Triangle A B Q, and so of all the rest, its Basis A B being 600, and its Perpendicular Q ✕ 290; multiply A B 600, by 145 half Q ✕, and you will have 87000 of plain Superfice, which write down as in the Book. In like manner I proceed with the Triangle Q B L, *viz.* by multiplying Q L 698 by 170, half B R its Perpendicular, so gets 98660 of Superficies, which place under the Superficies last found, and so on with the rest, till all the other Superficies be found; and being summed up, amount to 5,32561, being 5 Acres 32561; and that multiplied by 4, gives 1,30244, which

which is one Rood 30244; which multiplied by 40, produces 12,09760, making 12 Perches, rejecting 09760 its fractional Part, as useless. By this means having obtain'd the superficial Content of the whole given Figure,

Gather all the particular Sums, and set them down regularly under each other, as under the Figure marked A. for Acres, R. for Roods, and P. for Perches; the total Sum of which amounting to 6 Acres, 2 Roods, and 25 Perches, is the true superficial Content of the given Fig. 1. which was required; the like of all others.

PROP. IV.

How by the INFALLIBLE to survey the same Field, or Plot of Ground.

THE Instruments for surveying most in Use in *England*, are the plain Table, Circumferentor, Mr. *Gunter*'s Chain, and Theodolet. All which being so subject to Error, that scarce (with any of them) can we shut a Figure with any Certitude. Therefore

I betake my self to the *INFALLIBLE*, an Instrument of my own Invention, and so nam'd it, having by Experience found it the least fallible of all other Geometrical Instruments. And therewith to survey the foresaid Plot Fig. 1. do as followeth.

Let your firſt Station be at B, and from thence directing your Sights to A and L, draw along the Edge of each Ruler a ſtreight Line with your Lead Pencil, repreſenting the firſt and ſecond Side.

Now with the Chain meaſure thoſe two Sides, and you will find 600 for the firſt, and 555 for the Length of the ſecond Side; which mark upon the Inſtrument, *viz.* 600 along Line 1. and 555 along Line 2.

Then remove your Inſtrument to L, and from thence in manner aforeſaid direct your viſual Rulers to B and M, draw the Line 3 along the Edge of the Ruler pointing to M; ſo with your Chain having meaſured the Diſtance L M, find 430, which write down along the Line marked 3.

So changing your Station to M, where, as aforeſaid, directing your Sights to L and N, draw the Line 4 along the Edge directed upon N; whoſe Length, *viz.* 364, mark down along Line 4.

Then coming to N, where directing your Sights upon M and O, along the Ruler pointing to O, draw the Line 5; then O N meaſured, is found in Length 293, which mark upon your Inſtrument along the Line 5.

Which brings you to O, where directing your Sights to N and P, draw Line 6 along the Edge of the Ruler directed upon P, and O P meaſured being 530, mark it along the Line 6.

Now

Now with your Station at P, through your viſual Sights directed to O and Q, along the Ruler pointing to Q, draw Line 7, upon which write 475, the Length P Q.

Laſtly, From your Station at Q, looking as aforeſaid to the Objects P and A, draw along the Edge of the Ruler directed to A, Line 8; upon which write 409, the meaſured Diſtance betwixt A and Q. So have you all the Sides of the given Field Fig. 1. upon your Inſtrument; from which to transfer the ſaid Lines, and ſo angularly to joyn them, that they form the true and exact Figure (in all Reſpects) of the obſerved Field, do as followeth.

First take down your Inſtrument, and unfold the Paper of Obſervation; then with your Parallels applied to Line 1, transfer it to Fig. 2. and draw the Line AB; to which from your Decimal Scale give 600 in Length, equal to the firſt Side of the obſerved Field.

Then applying your Parallels along Line 2, transfer it to B, and draw BL, upon which ſet off 555, repreſenting the ſecond Side of the Field.

In like manner transfer Line 3 to Point L, where draw LM, in Length 430, which repreſents the third Side of the Field.

Again applying your parallel Rulers to Line 4, transfer it to M, where draw MN, in Length 364, repreſenting the fourth Side of the given Field. And

And thus transfer Line 5 to N, ſo draw N O, in Length 293, equal to the fifth Side of the foreſaid Field.

Further, Applying your Parallels to Line 6, transfer it to O, where draw O P, in Length 530, equal to the ſixth Side of the foreſaid Field.

Item. Transfer Line 7 to P, and draw PQ in Length, 475 to repreſent the ſeventh Side of the obſerved Field.

Laſtly, in manner aforeſaid transfer Line 8 to Q, and draw Q A, which cloſes the Figure; and being meaſured gives 409, equal to the Length of the eighth and laſt Side of the given Field, which infallibly proves Fig. 2. to be an exact Plan of Fig. 1. the Plot ſurveyed, which was required; the reſt by Inſpection.

PROP. V.

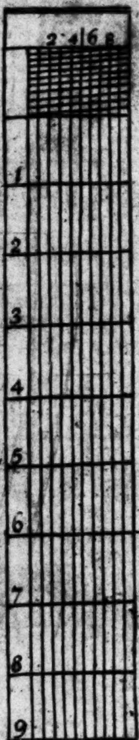
Of a Piece of Ground, in Length any Number of Roods; I demand what Number of Roods in Breadth makes up one Acre thereof.

THE Rule is, divide 160, the Number of Roods in one Acre ſquare, by its propoſed Length, and the Quotient will give the Breadth thereof.

For Example: Suppoſe a Piece of Ground to be meaſured, were 32 Roods in Length,

THE FIELD BOOK

A SCALE OF 1000 EQUAL PARTS, FOR FIG. 1



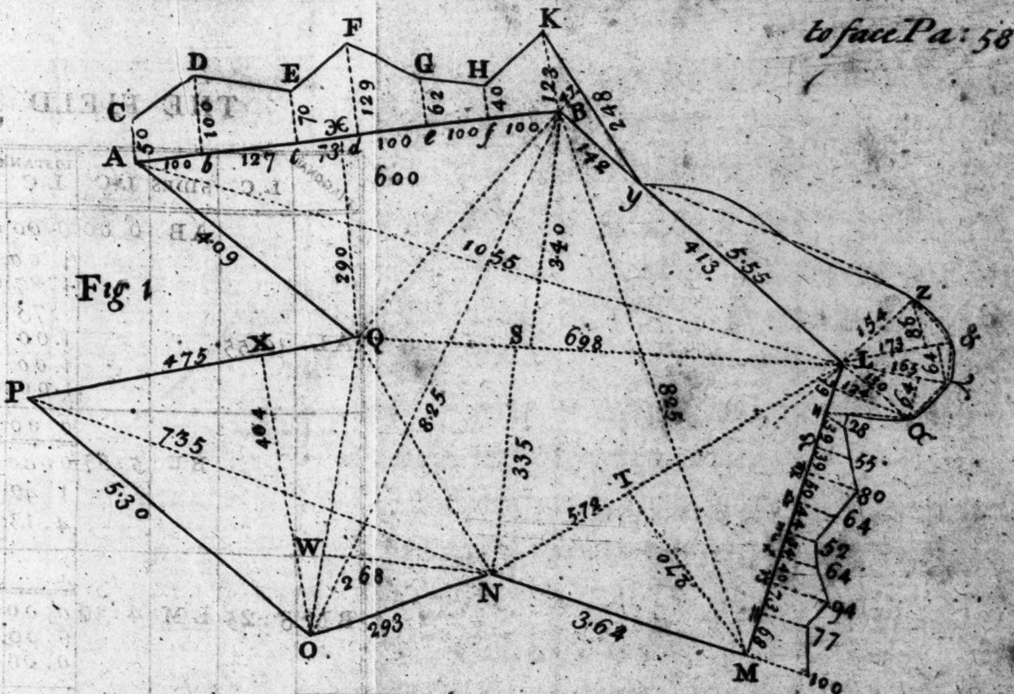
DIAGONAL	L.C	SIDES	L.C	DISTAN L C	OFFSET L C	CONTINGENCIES
AL.	10.55	AB	6 00	0.00 1.00 1.27 .73 1.00 1.00 1.00	0.50 1.00 0.70 1.29 .60 .40 1.23	574 } 82 77 } 600 49200 4 R^d 1.96800 40 p^r. 38.72000
		BL	5.55	6 00 0.00 1.42 4.13	5 74 .47 touch 1.54	
BM	8.25	LM	4.30	0.00 0.00 0.00 .79 .39 .39 .50 .44 .34 .40 .37 .68	1.73 1.65 1.30 .28 .55 .80 .64 .52 .64 .94 .77 1.00	430 from 79 Subst 351 Basis 7 } 351 614 } 68 99 } 2808 2106 23868 4 95472 40 38.18880
LN BO PN OQ	5.72 8.25 7.35 4.24	MN NO OP PQ QA	3.64 2.93 5.30 4.75 4.09	4.30	6.14	z&.......94 &v.......68 v♉.......83 } polig=ons

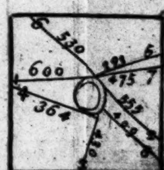
Angles	
KBy	....
yZL	....
LZ&	....
L&v	....
Lv♉	....
L♊	....

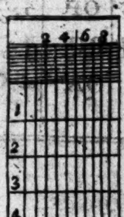
Roods.......

perches.......

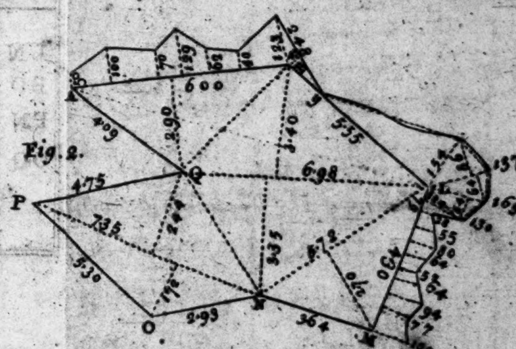
Fig 1







A SCALE OF 500 EQUAL PARTS



Angles	Superficies
KBy	0. 5828
yZL	3. 1801
LZ&	0. 7439
L&♈	0. 5536
L♈♉	0. 5280
L♉♊	0. 4898
	6 0782
	4
Roods	2·43128
	40
perches	17·25120

Angles	Superficies
ABQ	0. 87000
BLQ	0. 98660
LMN	0. 77220
LNQ	1. 16915
NOQ	0. 56816
POQ	0. 95950
Acres	5. 32561
	4
Roods	1. 30244
	40
perches	12·09760

	A^c. R^d. P^r
AB with offsets	00: 01: 38
BL with offsets and Little triangles	00:02: 17
LM with offsets	00:00: 38
all ye great Triangles	05:01: 12
Total	6: 02: 25

4 Roods makes 1 Acre

40 Perches makes 1 Rood

I demand how much thereof in Breadth is required to make one Acre.

OPERATION.

$$\left.\begin{array}{r} \cancel{1} \\ \cancel{160} \\ \cancel{32} \end{array}\right\} 5 \text{ Roods facit.}$$

Therefore 5 Roods in Breadth is equal to that Field, or Plot of Ground, which is 32 Roods long; or, which is all one, a Field of 32 Roods in Length, and 5 Roods in Breadth, is equal to one Acre.

Or, if you multiply 32 Roods by 5, you ſhall find 160 Product, the Number of Roods in one ſquare Acre.

$$\begin{array}{r} 32 \\ 5 \\ \hline 160 \end{array}$$

And thus is the following Table compoſed, where by Inſpection you will find the Solution of all ſuch Queſtions.

LEN.

LEN.	BREAD.	LEN.	BREAD.	LEN.	BREAD.	LEN.	BREAD.
40	4	30	$5\frac{1}{3}$	20	8	10	16
39	$4\frac{4}{39}$	29	$5\frac{5}{29}$	19	$8\frac{8}{19}$	9	$17\frac{7}{9}$
38	$4\frac{4}{19}$	28	$5\frac{5}{7}$	18	$8\frac{8}{9}$	8	20
37	$4\frac{12}{37}$	27	$5\frac{25}{27}$	17	$9\frac{7}{17}$	7	$22\frac{6}{7}$
36	$4\frac{4}{9}$	26	$6\frac{2}{13}$	16	10	6	$26\frac{2}{3}$
35	$4\frac{4}{7}$	25	$6\frac{2}{5}$	15	$10\frac{2}{3}$	5	32
34	$4\frac{12}{17}$	24	$6\frac{2}{3}$	14	$11\frac{3}{7}$	4	40
33	$4\frac{28}{33}$	23	$6\frac{22}{23}$	13	$12\frac{4}{13}$	3	$53\frac{1}{3}$
32	5	22	$7\frac{3}{11}$	12	$13\frac{1}{3}$	2	80
31	$5\frac{5}{31}$	21	$7\frac{13}{21}$	11	$14\frac{6}{11}$	1	160

An EXAMPLE *of the Table.*

Admit the Field to be meaſured were 33 Roods in Length, and required the Breadth to make one Acre.

Firſt in the Column marked LEN. for Length, find 33, and in the next Column oppoſite thereto, under BREAD. for Breadth, you ſhall find $4\frac{28}{33}$ for the required Breadth: That is to ſay, a Field of 33 Roods long, and $4\frac{28}{33}$ in Breadth, is equal to one Acre.

Another

Another Example. Suppoſe the Field to be meaſured were 20 Roods in Length, and required what Breadth there muſt be to make up one Acre: Look in the Table for 20, under the Column LEN. and oppoſite thereto you ſhall find 8, under the Column BREAD. which is the required Breadth; or if you divide 160 by 20, you'll find 8 in the Quotient. But if the Length be greater than any contained in the Table, the correſpondent Breadth to one Acre thereof ſhall be found Arithmetically as aforeſaid in the Rule. Or take ſome aliquot Parts, as an half, one third, or one fourth of the given Length, and thereby you may alſo come to your Purpoſe.

For Example: Suppoſe a Field to be meaſured were 80 Roods in Length, the Breadth required to make one Acre. Here, becauſe I find no Number ſo great in the Table under the Columns of Lengths, I divide 160 by 80, and ſo get 2 in the Quotient, for the required Breadth; or by ſome aliquot Parts of 80, as ſuppoſe 40, being the half thereof, which found in the Table gives 4 in Breadth, and the half thereof, *viz.* 2, is the Breadth required. If I had taken 20, the fourth Part of 80, I would have got 8 for Breadth; the one fourth whereof, *viz.* 2, is the required as before.

Or ſuppoſe a Field to be meaſured were 100 Roods in Length, and the Breadth required to make up one Acre. Divide 160 by

100,

100, and you will find $1\frac{3}{5}$ in the Quotient, equal to one Rood, 24 Perches, and that is the Breadth required. Or if I choose any aliquot Parts of 100, as 20 its fifth Part, I ſeek out for 20, under Column LEN. and finding 8 oppoſite thereto under Column BREAD. I take the one fourth Part thereof, *viz.* $1\frac{3}{5}$ for the true Breadth required. And in like manner I have calculated the following Table ſuitable to the *French* Meaſure.

In *France*, the Royal Foot, or *Pied du Roy*, is the longeſt, and always made Uſe of among the Mathematicians; I ſay the longeſt, becauſe there is ſcarce a City in *France*, *Flanders*, or *Brabant*, that differs not in their Foot, and other Meaſures; but the King's Foot is over all the ſame throughout the whole Kingdom.

In which Foot there are 12 Inches, and 6 Feet in every Toiſe. Again, there are 22 Feet in every current Perch; although in the Provoſtſhip of *Paris*, one Perch conſiſts of but 3 Toiſe, or 18 Foot. But the following Table is calculated according to the current Perch, or 22 Foot ſquare; ſo the Square of 22, *viz.* 484, I place in the laſt Column under BREAD. oppoſite to 1, under Column LEN.

Again, I divide 484 by 2, which gives 242, therefore write it as in the Table oppoſite to 2; then dividing 484 by 3, I have $161\frac{1}{3}$; which I place oppoſite to 3. Again, I divide 484 by 4,

and

and fo continue dividing 484 by all the reft of the Numbers up to 60, which finifhes the Table; and might go on further, but 60 may fuffice for any Ufe.

Len.	Bread.	Len.	Bread.	Len.	Bread.	Len.	Bread.	Len.	Bread.
60	$8\frac{1}{15}$	50	$9\frac{14}{25}$	40	$12\frac{1}{10}$	30	$16\frac{2}{15}$	20	$24\frac{1}{5}$	10	$48\frac{2}{5}$
59	$8\frac{12}{59}$	49	$10\frac{14}{49}$	39	$12\frac{16}{34}$	29	$16\frac{20}{29}$	19	$25\frac{9}{19}$	9	$53\frac{7}{9}$
58	$8\frac{10}{29}$	48	$10\frac{1}{12}$	38	$12\frac{14}{19}$	28	$17\frac{2}{7}$	18	$26\frac{8}{9}$	8	$60\frac{1}{2}$
57	$8\frac{28}{57}$	47	$10\frac{14}{47}$	37	$13\frac{3}{37}$	27	$17\frac{25}{27}$	17	$28\frac{8}{17}$	7	$69\frac{1}{7}$
56	$8\frac{9}{19}$	46	$10\frac{12}{23}$	36	$13\frac{4}{9}$	26	$18\frac{8}{13}$	16	$30\frac{1}{2}$	6	$80\frac{2}{3}$
55	$8\frac{44}{55}$	45	$10\frac{34}{45}$	35	$13\frac{29}{35}$	25	$19\frac{9}{25}$	15	$32\frac{4}{15}$	5	$96\frac{4}{5}$
54	$8\frac{26}{27}$	44	11	34	$14\frac{3}{17}$	24	$20\frac{1}{6}$	14	$34\frac{4}{7}$	4	121
53	$9\frac{7}{53}$	43	$11\frac{11}{43}$	33	$14\frac{22}{33}$	23	$21\frac{1}{23}$	13	$37\frac{3}{13}$	3	$161\frac{1}{3}$
52	$9\frac{4}{13}$	42	$11\frac{11}{21}$	32	$15\frac{1}{8}$	22	22	12	$40\frac{1}{3}$	2	242
51	$9\frac{25}{51}$	41	$11\frac{33}{41}$	31	$15\frac{19}{31}$	21	$23\frac{1}{21}$	11	444	1	484

An

An EXAMPLE *of the Table.*

Admit a Piece of Ground to be meaſured were 22 *French* Feet in Length, and required what Breadth will make one Square Perch.

Firſt, In the Column marked LEN. find 22. and oppoſite thereto, under the Column marked BREAD. ſtands 22; that is to ſay, 22 Feet in Length, and 22 in Breadth, makes one Perch; or divide 484 by 22, and you ſhall find 22 in the Quotient for the required Breadth.

Again, Suppoſe a Plot or Field, of 44 in Length, and required what Breadth thereto will make up one Square Perch.

In the Table under LEN. find 44, and under BREAD. oppoſite thereto, you'll meet with 11, and that's the required Breadth; or divide 484 by 44, and you'll get 11 in the Quotient.

Laſtly, Admit a Piece of Ground to be meaſured were 48 Feet in Length, and required what Breadth thereof will make up one Square Perch.

Under Column LEN. find 48, and oppoſite thereto, under Column BREAD. you will find $10\frac{1}{12}$, or divide 484 by 48, and the Quotient will give $10\ \frac{4}{48} \Big| \frac{2}{24} \Big| \frac{1}{12}$.

Therefore I ſay $10\frac{1}{12}$ Feet is the required Length, the like for all other Queſtions of this Nature.

OPE-

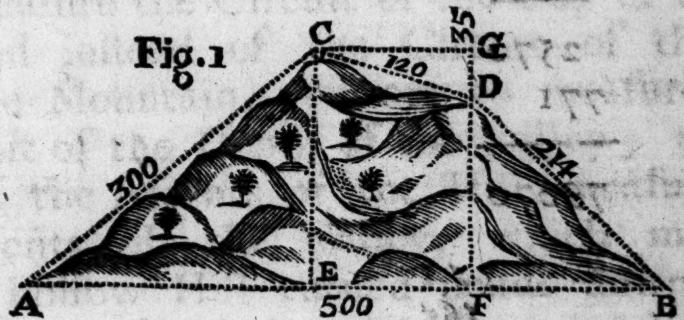
OPERATION.

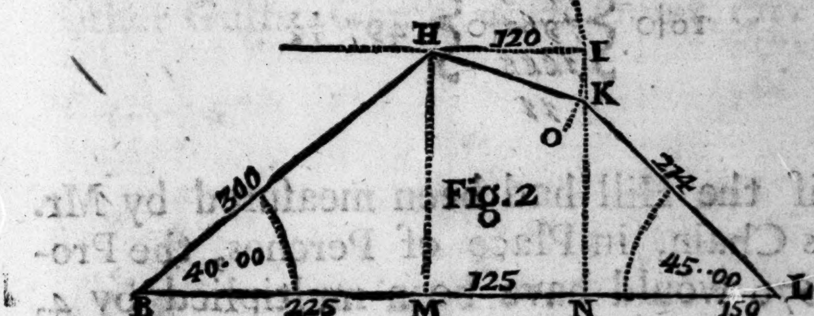
$$\left.\begin{array}{l} \cancel{4}\,8\,(4 \\ \cancel{4}\,\cancel{8}\;8 \end{array}\right\} 10\tfrac{4}{48} \;\Big|\; \tfrac{2}{24} \;\Big|\; \tfrac{1}{12}.$$
$$\quad\cancel{4}$$

PROP. VI.

To take the superficial Content of a Hill, as also to find its Horizontal Line.

SUppose ACDB, Fig. 1. to be the Mountain, and the Circuit of its Basis AB 500 Perches; which known, Measure the Circumference of its Top CD, suppose 120 Perches, which added to 500, make 620 Perches.





 The

The measure of the Ascent of the Hill AC 300, with the opposite Ascent 214 Perches, and adding those two together makes 514, the half of which is 257; and that multiplied by half the Sum of the upper and under Circuits of the Mountain, *viz.* 310, gives 79670 of Product, which divided by 160, the Number of square Perches in one Acre, gives 497 $\frac{15}{16}$ Acres in the Quotient, which is the superficial Content required.

OPERATION.

AC	300		AB	500	
BD	214		CD	120	
	514	257		620	310

257
310
———
2570
771
———
79670

$16|0 \left\{ \begin{array}{l} 11 \\ 165 \\ 3525 \\ 7967|0 \\ 1666 \\ 11 \end{array} \right\} 497 \frac{15}{16}$

But if the Hill had been measured by Mr. *Gunter*'s Chain, in Place of Perches, the Product 79670 would have been multiplied by 4, whereby

whereby to have found the Roods, and then by 40, to have found the Perches; and the Work would have stood as followeth.

Product	79670
	4
Roods	3.18680
	40
Perches	7.47200

Here the whole superficial Content of the Hill does not exceed 3 Roods 7 Perches.

To measure a Valley, in place of the Circuit of the Base of a Mountain, you must measure the Circuit of the Top of the Valley, and instead of the Circuit of the Top of the Mountain, you are to measure the Circuit of the Depth of the Valley; and instead of the Ascents of the Hill, measure the Descents of the Valley, which makes it a a hollow Hill turned upside down; and to be measured by the foregoing Rule, as all other Gulfs, Glens, deep Holes, &c.

How to take the Horizontal Line of a Hill.

IN Fig. 1. of the laſt Propoſition AB, ſhall be the Horizontal Line required; the which to find,

Firſt take the Angle EAC, 40° 00″ then the Angle FBD 45° 00″ which mark down in your Field Book, with the Aſcents of each Side, and Circuits at Top and Bottom.

Then upon a Sheet of clean Paper draw the Line BL, as in Fig. 2. at Random; and at B, make the Angle MBH, of 40 Degrees, equal to EAC Fig. 1. make BH 300 equal to AC, and from H let fall the Perpendicular HM.

Alſo with 120 from your decimal Scale, with one Foot in H, deſcribe the Arch O; but becauſe the Hill is higher at C, than at D, bring C and D to a Level, by placing the Picket DG Fig. 1. with its Top juſt in a Level with C: So will DG be 35 in Length, which likewiſe mark down in the Field Book. Draw HI in Fig. 2. parallel to MN. Let fall the Perpendicular IN, then from IN ſubſtract IK equal to DG, and there will remain KN, draw KL the Slope of the Hill, which meaſured will be 214, equal to BD.

Thus

Thus having finiſhed the Work, meaſure BM, and you will find it 225 in Length, MN 125, and NL 150; all which added together make up 500; for the Length BL equal to AB, the Horizontal Line of the Hill Fig. 1. which was required.

I ſhall conclude this Treatiſe by adding a Proof of the whole Work; which is, in rendering upon Paper a true and exact Plan of any Plot of Ground ſurveyed in the Fields.

For Example: Let Fig. 1. be the Plot of Ground to be ſurveyed as followeth. With your Inſtrument take your firſt Station at B, where directing your Sights to A and C, draw along the Edges of the two Rulers the Lines 1 and 2, ſignifying the firſt and ſecond Side, which meaſured with the decimal Chain, is found in Length 412 for the firſt Side, and 193 for the ſecond. Which done,

Remove your Station to C, and from thence directing your two Sights, the one to B, the other to D, draw the Line marked 3, which meaſured with your Chain, is in Length 288, for the third Side.

Remove now your Station to D, and there directing your Sights, the one upon C, the other upon E, draw the Line upon the Inſtrument marked 4; which meaſured as aforeſaid gives 293 for the fourth Side.

Remove again your Inſtrument to E, where directing your Sights to D and F, draw upon

your Inſtrument the Line 5; which meaſured with the Chain gives 192 for the Length of the fifth Side.

Remove your Sation again to F, where direct your Sights, the one to E, with the other upon G; and ſo along the Edge of the Ruler directed on G, draw Line 6, which meaſured by the Chain gives 262 for the ſixth Side.

Laſtly, Removing to G, direct your Sights the one upon F, the other on A, the firſt Station, and ſo along the Edge pointing to A, draw upon the Inſtrument the Line 7; and that meaſured with the Chain gives 289 for the Length of the laſt Side; all which Sides, each in their due Lengths, being diſtinctly marked on their reſpective Lines upon the Inſtrument, or Paper of Obſervation, remains only (by Help of your parallel Rulers) to transfer thoſe Lines to ſome other convenient Place on the Paper, and thereby form the exact Plan of Fig. 1. the Field or Plot of Ground which was ſurveyed; to perform which do as followeth:

Having taken down your Inſtrument, and unfolded the Paper of Obſervations, lay the Edge of your Parallels to Line 1, and ſo transfer it to any Part on the Paper at Option; but here becauſe I have Place enough, upon the ſame Line prolonged I draw the Line AB, Fig. 2. and from the Decimal Scale

Scale thereto annexed, I take 412, wherewith I terminate the Length of A B, repreſenting the firſt Side of the Field.

Here looking to the Figure of the ſurveyed Plot, (which you ought to have by you upon a Paper apart) obſerve to what Hand you went in going round the Field; and accordingly with your parallel Rulers applied to Line 2, transfer it to Point B, and ſo draw Line BC; to which (from the ſame Scale) give 193 for Length.

And now we are to prove the Work as we go along, and for that End, upon the Paper of Obſervation, prolong Line 1 and 2, till the Interſect in H; then from H, deſcribe the Arch comprehending that Angle, which ſhall be equal to Angle B.

Then with your Parallels as aforeſaid transfer Line 3 to Point C, and draw C D; on which (from your Scale) ſet off 288, for the Length of that Side; and upon the Paper of Obſervation prolong Line 3, till it interſects Line 2 at 1; and from the Point 1 deſcribe the Arch comprehending that Angle, and will be found equal to that at C.

Again, with your Parallels transfer Line 4 to the Point D, ſo draw D E, to which give 293, repreſenting the fourth Side; and as aforeſaid prolong upon the Inſtrument the Line marked 4, and it will interſect that marked 3 in the Point K; from which deſcribe

 the

the Arch comprehending the Angle at K, which ſhall be equal to Angle D.

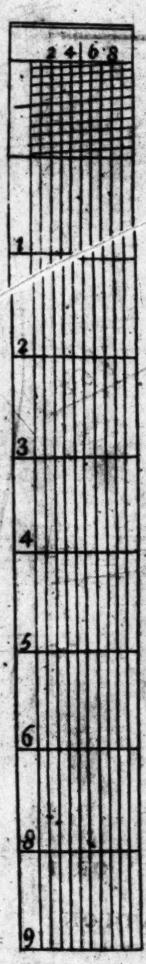
Further, from your Inſtrument in manner aforeſaid, transfer the Line 5 to Point E, and draw E F, in Length 192, for the fifth Side. Next upon your Paper of Obſervation prolong Line 5, till it interſect Line 4 at L; and from L deſcribe the Arch comprehending that Angle, which ſhall be equal to that at E.

Proceed yet in transferring Line 6 to Point F, and there drawing F G, make that Line in Length 262, repreſenting the ſixth Side of the Field; and ſo prolonging Line 6 till it interſects Line 5 at M; from thence deſcribe the Arch comprehending that Angle, will be found equal to Angle F.

Laſtly, Transfer Line 7 to Point G, and draw A G; which meaſured upon your Decimal Scale here annexed, is juſt 289, as found upon the Field by the Chain, proves the Work to be true with the Infallibility of my Inſtrument.

But yet for a compleat Illuſtration, prolong upon the Paper of Obſervation Line 7, till its Interſection of Line 6 at N, and from that Point deſcribe the Arch comprehending that Angle, which ſhall be found equal to the Angle at G.

In like manner if you prolong Line 7 upon the Inſtrument, it will interſect Line 1 in the Point O; from which deſcribe the occult Arch comprehending that Angle, which meaſured



THE HALF OF A RHINLAND'S FOOT.

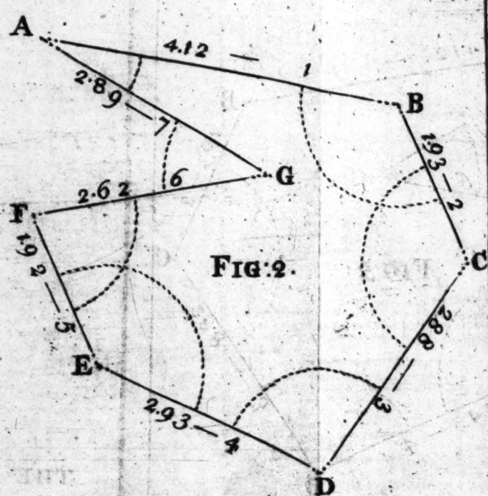
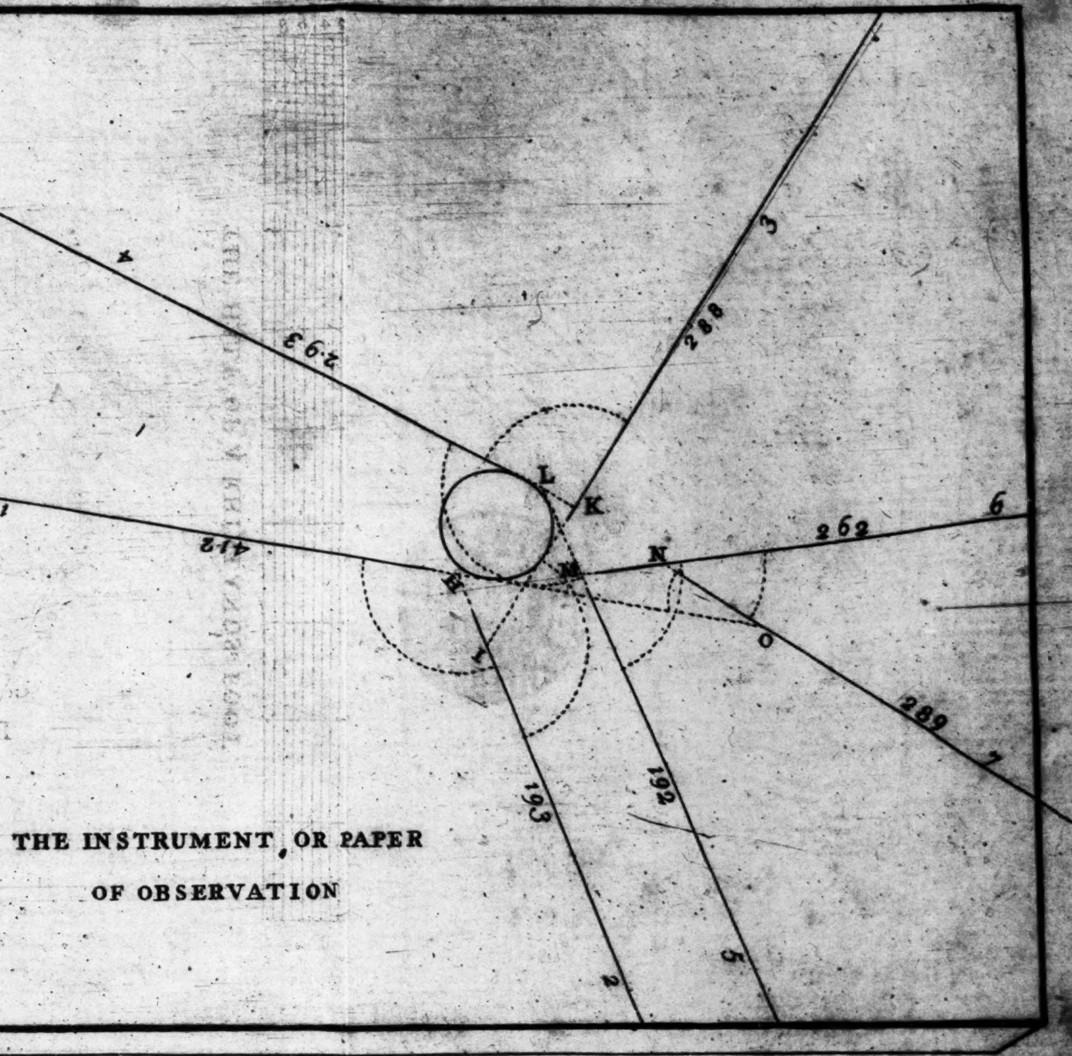


FIG. 2.

after Pa: 72



THE INSTRUMENT, OR PAPER OF OBSERVATION

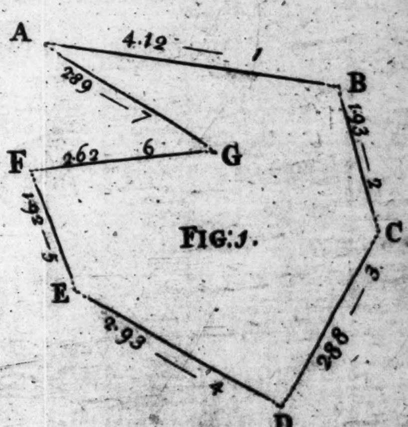


FIG: 1.

ſured will be found equal to the Angle at A.

So giving you this for a practical and demonſtrative Proof of the whole Work, I ſhall need no longer to inſiſt on this Subject.

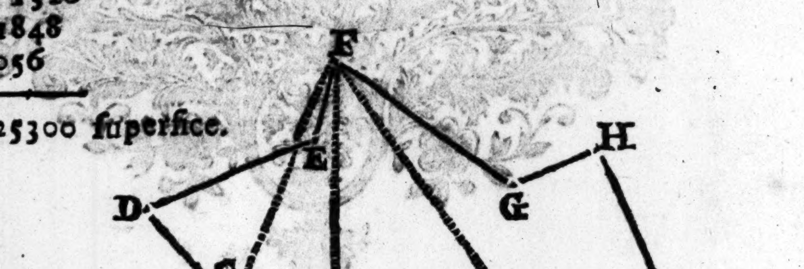
APPEN-

APPENDIX.

dk 528 } 264
475 FM

1320
1848
1056

125300 superfice.



How to reduce all Sorts of plain Figures to one Triangle, equal in Superfice to the given Plane.

BY this Propoſition the Surveyors may avoid the Trouble of throwing many angular Figures into ſeveral Triangles, as formerly, in order to calculate the ſuperficial Content, as ſhall appear by the following Operation.

Firſt produce A L, one of the Sides of the given Figure, to any convenient Length, for the Baſis of a required Triangle, which ſhall be equal in Superfice with the given Figure.

You may begin the Operation either on the right or left Hand of the Figure, as here I chooſe the left, where I ſee the firſt ſalient Angle is A B C; therefore from A and C, the extream Points of its Legs, I apply my Parallels, and from thence opening them to B, the ſalient Point of the Angle, I interſect the Baſis at a.

Secondly, From a, the Point laſt found, and D the next ſalient Angle, applying the Edge of your Parallels, open them to C, the entering Angle, and the Edge of the Ruler will cut the Baſis at b; then from b, the Point laſt found, and E, the Angle next to be handled, laying the Edge of your Parallels, open them to D, and thus you'll interſect the Baſis at c.

Thirdly,

Thirdly, From c, the Point laſt found, applying the Edge of your Rulers to F, the Angle now in its Tour to be made uſe of, and ſo opening them to E, the laſt entering Angle, will cut the Baſis in the Point d, draw the Line d F, which is the left Side of the required Triangle.

Now for the right Hand Side of the Figure: Firſt, As in the left Hand Operation, lay the Edge of your Parallels from L to I, the Extremities of the Legs of the Angle at K, and from thence opening to the angular Point K, mark the Baſis at f; then from f, laying the Edge of your Rulers to H, open again to I, ſo ſhall you interſect the Baſis in g.

Secondly, Your Edge laid from g to G, open your Parallels to H, and you will cut the Baſis in h.

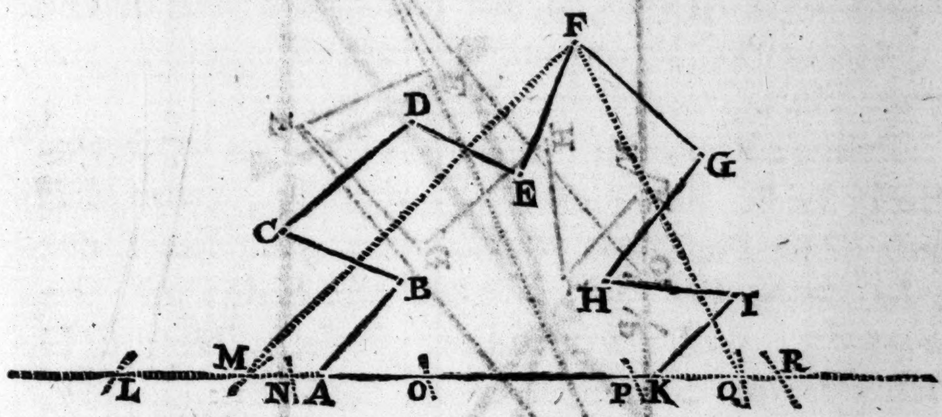
Thirdly, From h, the Point laſt found, and F, laying the Edge of your Rulers, open them to the angular Point G, and you will cut the Baſis in k.

Laſtly, Draw the Line F k, and it gives the required Angle d k F, equal in Superfice to the given Figure.

The Demonſtration of this is proved from the Theorem in the firſt Book of *Euclid*, that Parallelograms and Triangles, having the ſame Baſis, and between the ſame Parallels, are equal.

So that the Superfice of this Triangle d k F, being taken after the ordinary Way, whoſe Baſis

Basis dk is 528, and perpendicular FM 475 of those equal Parts; whereof AL, the Side of the Figure hath 146, will be found 125300 equal to the whole Superfice of the given Figure, as thereto annex'd.

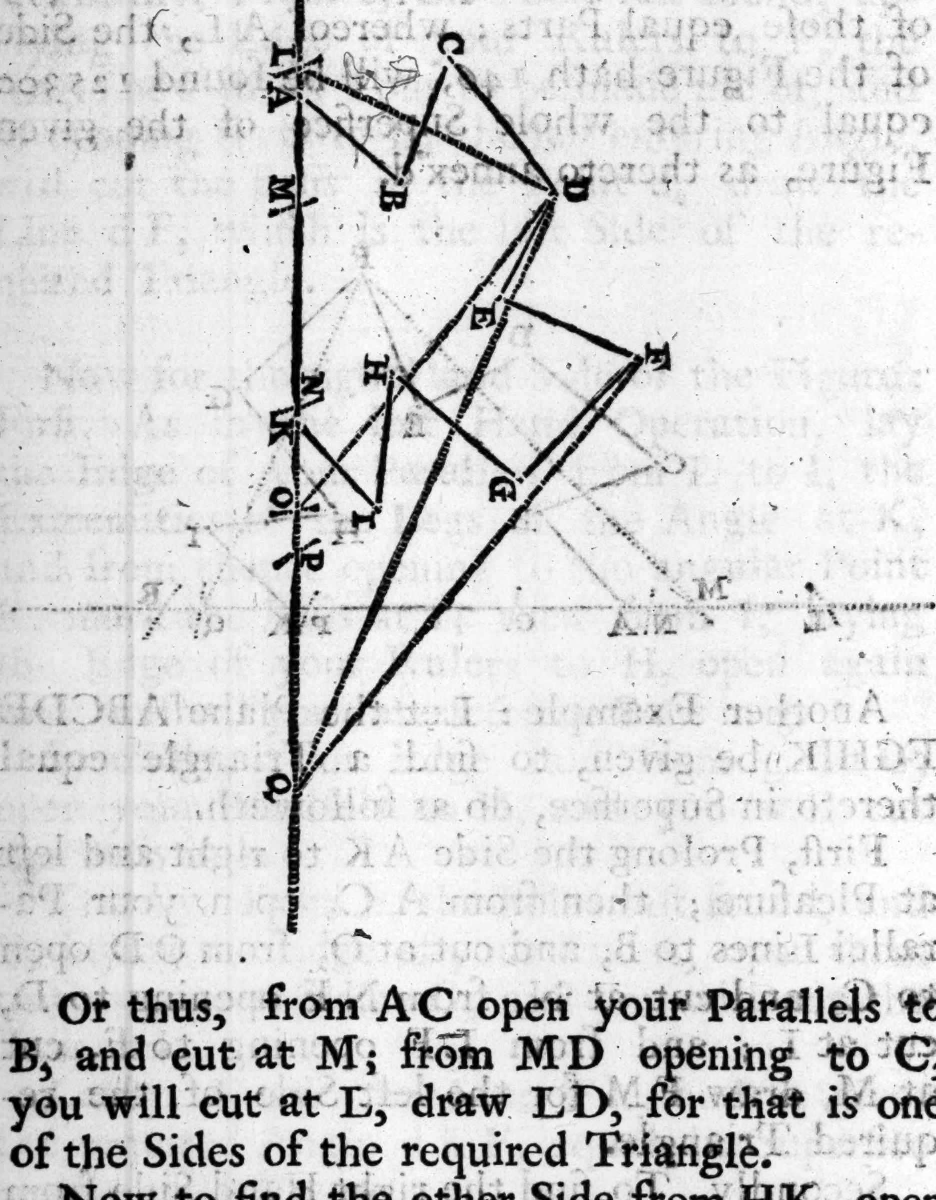


Another Example: Let the Plane ABCDE FGHIK be given, to find a Triangle equal thereto in Superfice, do as followeth.

First, Prolong the Side AK to right and left at Pleasure; then from A C, open your Parallel Lines to B, and cut at O, from O D open to C, and cut at N, from N E opening to D, cut at L; and from L F opening to E, cut at M, draw F M for the left Side of the required Triangle.

Secondly, To find the right Hand Side from KH, open your Rulers to I, and cut at R; from R to G, opening to H, cuts at P, and from P F opening to G, cuts at Q, draw F Q; and that is the left Side required: So have you the Triangle MFQ, equal in Superfice to the given Plane, which was required.

Or

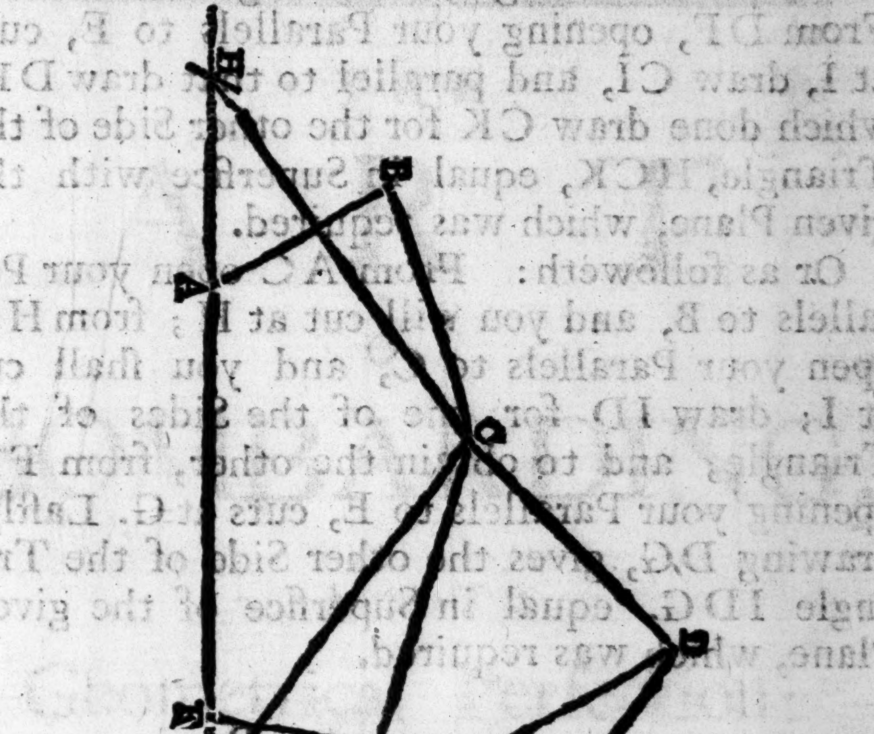


Or thus, from AC open your Parallels to B, and cut at M; from MD opening to C, you will cut at L, draw LD, for that is one of the Sides of the required Triangle.

Now to find the other Side from HK, open your Parallels at I, and cut at P, from PG opening to H, cuts at N, from NF opening to G, cuts at O, draw OD; and parallel to that draw FG, and it will cut in Q. Lastly, Draw QD, for that is the Hypothenus of the Triangle LDQ, equal in Superfice of the above given Plane, which was required.

Another

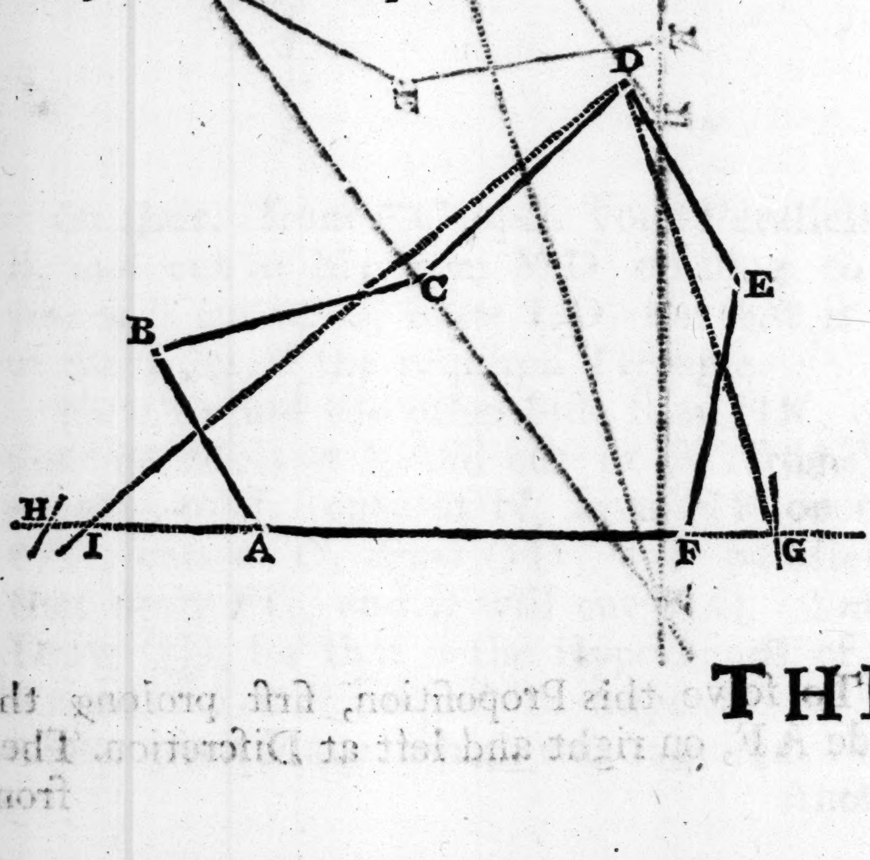
Another Example, where the Plane ABC DEF is given, and required a Triangle, which shall be equal in Superfice thereto.



To solve this Proposition, first prolong the Side AF, on right and left at Discretion. Then from

from AC, opening your parallel Rulers to B, cuts at H, so draw HC, one of the Sides of the required Triangle, and to get the other. From DF, opening your Parallels to E, cuts at I, draw CI, and parallel to that draw DK, which done draw CK for the other Side of the Triangle, HCK, equal in Superfice with the given Plane, which was required.

Or as followeth: From AC open your Parallels to B, and you will cut at H; from HD open your Parallels to C, and you shall cut at I; draw ID for one of the Sides of the Triangle; and to obtain the other, from FD opening your Parallels to E, cuts at G. Lastly, drawing DG, gives the other Side of the Triangle IDG, equal in Superfice of the given Plane, which was required.



THE

THE

ART

OF

BOMBARDING,

IN ITS

Geometrical Perfection:

Clearly ſhewing how by the Square and Compaſs, to deſcribe all Sorts of Parabola's ſhot from the Elevation of any Degree in the Quadrant.

Dedicated to His Majeſty's Gunners in the TOWER.

FOR Example: Let it be required to deſcribe the Parabolick Shot from the Elevation of 45 Degrees, the fartheſt Flight whereof, or utmoſt Random, ſuppoſed 100 equal Parts. Fig. 1.

I. Draw AB, to repreſent the Horizontal Line, and furtheſt Shot, conſiſting of 100 equal

equal Parts, the which divide and number off as you fee; from the middle Point thereof erect the Perpendicular NP, with the Semi-diameter NP, taken equal to BN; from N describe the Semi-circle APB, and divide it into 45 Degrees; which done, from B extending your Compasses first to $\frac{40}{50}$, then to $\frac{30}{60}$: to $\frac{20}{70}$: lastly to $\frac{10}{80}$; describe obscure Arches, till they touch the Horizon, will thereupon denote the Length of the Flight of the Bomb or Ball, shot from the Elevation of any of these Degrees: As the Arch of $\frac{30}{60}$ falling in the Point C, gives near 87 upon the Horizon, for the Length of its Flight BC, and the like for all the rest of the occult or obscure Arches. Now for the Parabola intended.

From R, half NB, make the Angle NRO of 45 Degrees, giving NO for the Height of the Parabola; from N up to P describe any Number of Lines you please, but all parallel to AB, as in this Example 10, marked on the right 1, 2, 3, 4, &c.

Take the Distance SP in your Compasses, and with one Foot in N (now your Point of Equality) touch the first Parallel on right and left, in the Points 1—1. Again, with the Extent TP, from N as aforesaid, touch the second Parallel in the Points 2—2: So with VP from N, touch the third Parallel in the Points 3—3. Lastly, with the Extent XP, from N as aforesaid, touch the fourth Parallel in the Points 4—4; and thus have you all the Points required, which if you trace along from

rom Point to Point with an unſhaken Hand, you will form the parabolical Line A O B, which was required. And if you draw the obſcure Line P B, it will be parallel to O R, and a Tangent to the Parabola B O A.

II. *To deſcribe the Parabolick Shot from the Elevation of 30 and 60 Degrees.*

Divide B C the fartheſt Flight thereof into two equal Parts at E, where raiſe the Perpendicular E F; from H, half E B, make the Angle E H D of 60 Degrees; and the Angle E H K of 30°, its Comp. giving E K for the Difference of the Parameter, and E D the Height of the Parabola.

From D, ſet the Difference of the Parameter to F and G; which done with the Extent ♍ F, ſetting one Foot in G, (now the Point of Equality) with the other touch the firſt Parallel on right and left, and mark the Interſections thus ˙˙.. Again, with the Diſtance ♎ F from G, mark the ſecond Parallel on right and left ˙˙. as the former. Then with ♏ F from G, mark the third Parallel as aforeſaid; and with ♐ F from G as aforeſaid, mark the fourth Parallel: with the Diſtance ♑ F from G; mark (as formerly) the fifth Parallel with ♒ F from G; mark the ſixth Parallel ˙˙. on right and left. Laſtly, with the Extent ♓ F from G, the Point of Equality in manner aforeſaid, mark the ſeventh Parallel ˙˙. on right and left, ſo have you all the Points re-

 quired;

quired; which as aforesaid if you draw with an even Hand, you will have formed the Parabola C D B required, shot from the Elevation of 60 Degrees.

III. *To describe the Parabolick Shot from the Elevation of* 30 *Degrees.*

Set E D from E to M, and E K from E to I, draw the obscure Lines H M and H K; so are the Angles E H M, and E H D equal; draw now the obscure Line H I, so is the Angles E H K and E H I equal.

Lay the Distance E I or E K, from M to L; so is L the Point of Equality; then with the Extent ♍F, from L, mark the first Parallel on right and left ∴ as formerly.

Lastly, With the Distance ♎F, with one Foot in L, with the other as aforesaid, mark the second Parallel, so have you all the Points required; which tracing with an even Hand from Point to Point, describes the Parabola CIB required.

IV. *To describe the Parabola of* 70 *and* 20 *Degrees.*

Upon the Point ♊ half its farthest Flight, or half B z, erect the Perpendicular ♊♉ prolonging it below the Horizon at pleasure; from ♈ half B ♊ make the Angle ♊♈♉ of 70 Degrees; and the Angle ♌♈♊ of 20 its Comp. giving ♊♉ for

for the Height of the Parabola, and ♊♌ for the Difference of the Parameter.

From ♉ ſet the Difference of the Parameter to ♋ and *a*; ſo is *a* the Point of Equality, and ♋ for the Top Point: Now placing all one Foot in *p*, extend the other to the Top Point ♋; and with that Extent ſetting one Foot in *a*, the Point of Equality, with the other interſect the firſt Parallel on right and left, which as formerly mark thus .·˙ Again, with *r* ♋, from *a* the Point of Equality as aforeſaid, interſect and mark the ſecond Parallel: *Item*, with *s* ♋ from *a*: Interſect as formerly the third Parallel: Alſo with *t* ♋ from *a* as aforeſaid, interſect and make the fourth Parallel on right and left; with *u* ♋ from *a* the Point of Equality, touch and mark the fifth Parallel with *x* ♋; from *a* as aforeſaid interſect and mark the ſixth Parallel with *y* ♋; from *a* interſect and mark the ſeventh Parallel. And laſtly, with *&* ♋ from *a* interſect the eighth Parallel on right and left; marking it .·˙ in manner aforeſaid; ſo have you all the Points required; which followed with an even Hand from point to point, leaves the Impreſſion of the Parabolick Shot from the Elevation of 70 Degrees, which was required.

V. *To deſcribe the Parabolick Shot from the Elevation of 20 Degrees.*

Make ♊ *b* equal to ♊♉, and ♊ W equal to ♊♌; ſo have you ♊ W for the Height of the

Parabola with the Angle ♊♈ *b*, equal to the Angle ♊♈♉, and the Angle ♊♈W equal to the Angle ♊♈♌. Set ♊ W, or ♊♌, from *b* to *d*, so is *d*, the Point of Equality. Now placing one Foot in *p*, extend the other to ♋; and with that Extent, with one Foot in *d*, the Point of Equality, touch the first Parallel on each Side marked thus .___. which are all the Points necessary, whereby as aforesaid to describe the required Parabol ZWB: But in case you think the Distance ♊ *p* too great, you may draw an intermediate obscure Parallel Line, whereby you may find the two other Points .___. on right and left, the one towards Z, the other near to B, which will be helpful in describing the Parabola, which you'll find in the Practice.

VI. *To describe the Parabolick Shot from* 80 *and* 10 *Degrees, and the like for all others.*

From *e*, half its Flight B *f*, erect the Perpendicular *e k*, and prolong it downwards at Discretion. On the Point *g*, half B *e*, make the Angle *e g l* of 80 Degrees, as also the Angle *q g e* of 10°. its Comp. Lay *q e*, from *e* to I, so have you *e l* for the Height of the great Parabola, and *e* I for the Height of the little one. Set the Length *eq*, from *l*, to *k* and *m*, giving *l* for the Top Point, and *m* the Point of Equality; which done proceed as formerly, *viz.* by setting first one Foot of your Compass in *l*, and extending the other to *k*; and

with

with that Length from *m*, the Point of Equality, touch and mark as aforeſaid the firſt Parallel on each Side thus .·˙ then with ☿ *l* as aforeſaid, from *m* the Point of Equality mark the ſecond Parallel with ♂ *l*, from *m* mark the third Parallel with ♃ *l*, from *m* mark the fourth Parallel with ☉ *l*, from *m* make the fifth Parallel with ♄ *l*, from *m* mark the ſixth Parallel with Æ *l*, from *m* mark the ſeventh Parallel with Q *l*, from *m* mark the eighth Parallel. Laſtly, With the Diſtance *w l* from *m*, the Point of Equality,(as hitherto done) mark the ninth Parallel on both Sides .·˙ —— So have you all the Interſections neceſſary, whereby to deſcribe the required Parabola *f l* B, by following from point to point as in the foregoing, and it's done.

VII. *To deſcribe the Parabola of* 10 *Degrees, Compliment of the former.*

Make *e n* equal in Length to *e l*, draw *g n* and *g* I, giving the Angle *e g n* equal to *e g l*, and *e g* I equal to the Angle *e g q*; from *n* lay off the Length *e q* to *h*, and that is now the Point of Equality. And now becauſe the Point I, denoting the Height of the required Parabola, is (by far) not ſo high as the firſt Parallel, betwixt I and the Horizon you may draw an intermediate obſcure Parallel, from the Interſection of which, with the Line *e l*, extend your Compaſſes to the Top point *k*; and with that Diſtance ſetting

one Foot of your Compaſs in *b*, the Point of Equality, mark the obſcure Parallel on each Side thus _.___. Laſtly, Following from the Point *f* to B, touching the ſaid two Points . I . you will have left the true Impreſſion of the required Parabola, and it's done.

Any Parabola being given, to find from what Degree of the Quadrant it was deſcribed.

The which to ſolve, and all ſuch like Queſtions, divide firſt the Baſe of the Parabola into two equal parts, where raiſe a Perpendicular, as E D in the foregoing Parabola, from the Elevation of $\frac{30}{60}$; again divide E B its other half into two equal parts at H, from which draw H D, ſo will you have the Angle E H D, which meaſured on a Line of Chords, will give 60 Degrees for that from which the given Parabola C D B was deſcribed.

How by the natural Sines to determine the Length of all the Flights, ſhot from the Elevation of any Degree in the Quadrant.

Example: Suppoſe firſt it were required to know the Length of the Bomb's Flight, made from the Elevation of 10 Degrees.

Firſt know, That the Sine of double the Angle of any Elevation, gives the true Length of the fartheſt Flight of the Bomb or Ball, (if the Elevation be under 45 Degrees, cal-

led *umbra versa*; but if the Elevation be above 45 Degrees, then called *umbra recta*, the Sign of double its Comp. gives the Length of the Flight.

So in this Example, where the Flight of 10 Degrees is required, see in the Book of natural Sines for the Sine of 20 Degrees, double the Angle of Elevation, and you will find 34.2 for the Length B*f*.

If the Flight from the Elevation of 80 Degrees had been required, it would give the same, because 20 Degrees twice 10 its Comp. sought for in the Book of Sines, gives 34.2 as before; which measured on the Horizon AB, will be found exactly to agree.

Again: Let it be required to find the Length the Mortar-piece will throw the Bomb, from the Elevation of 30 Degrees.

According to the foregoing Rule, find the Sine of 60 Degrees, double the Elevation, and you'll have 86.6 for the required Length or BC.

But if the Flight made from the Elevation of 60 Degrees had been required, you would have found the same Number in the Book of Sines, because you would have sought for twice 30 Degrees double the Com. of the Angle of Elevation.

Now supposing the furthest Shot from the Elevation of 45 Degrees, to be 50 Toyses, or equal Parts, I demand how far that Mortar-piece, with the same Weight of Ball, and

Quantity of Powder, will throw her Ball from the Elevation of 30 Degrees.

To ſolve this and the like Queſtions, ſay by the Rule of Three, as Rad. 100 to 50 Toyſes, ſo is 86, the Sign of double the Angle of Elevation to 43 Toyſes, or equal Parts, for the Length of the required Flight.

OPERATION.

100 —— 50 —— 86
50
——
facit —— 43|00

Again: The Queſtion may be ſtated thus: If from the Elevation of 30 Degrees, my Mortar throws 43 Toyſes, how far will ſhe throw from the Elevation of 45 Degrees? Anſwer, 50 Toyſes.

OPERATION.

3.60 TO. s.45
86 —— 43 —— 100
100
—— 4300 } 50 facit
866
8

This Method of working is ſtill to be obſerved, whilſt the known Diſtance or Flight exceeds not two Figures; but when it does, as in the following Example, where it conſiſts of

of three Figures, uſe then the firſt three Figures of double the Angle of Elevation, for the third Term of the Rule.

Example: If from the Elevation of 45 Degrees, a Mortar-piece throw her Bomb 150 equal Parts or Roods, I demand, how far will ſhe throw her Bomb with the ſame Quantity of Powder, from the Elevation of 20 Degrees? Anſwer, *96.3* equal Parts.

Note. Where the known Flight exceeds not two Figures, Radius muſt have three; when the known Flight conſiſts of three Figures, Radius muſt have four; obſerving ſtill for a Rule, that Radius muſt conſiſt of one Cypher more than the given Diſtance; and that the third Term of the Rule muſt conſiſt of the ſame Number of Figures as doth the ſecond Term.

OPERATION.

Rad. — 1000 — 150 — *642* s of 40° doub. the ∠
150 of Elevat.

———

32100
642

———

96.300 facit.

Therefore I ſay, if from the Elevation of 45 Degrees, a Mortar-piece throws her Bomb 150 equal Parts, ſhe will throw the ſame Weight of Bomb *96.3* equal Parts from the Elevation of 20 Degrees.

Laſtly,

Laftly, (to prove all) If from the Elevation of 20 Degrees, you throw a Bomb 96.3 equal Parts, or Roods, how far can you throw the fame Bomb from the Elevation of 45 Degrees? To folve this, fay, as 642 the Sign of double the given Angle of Elevation, to 963 its known Flight, fo Rad. 1000 to 150 equal Parts.

OPERATION.

642 —— 963 —— 1000

$$\left.\begin{array}{c} \cancel{3}\cancel{2}\cancel{1} \\ \cancel{9}\cancel{6}\cancel{3}\cancel{0}\cancel{0}\cancel{0} \\ \cancel{6}\cancel{4}\cancel{2}\cancel{2}\cancel{4} \\ \cancel{6}\cancel{4}\cancel{4} \\ \cancel{6} \end{array}\right\} 150 \text{ facit}$$

Another Method of defcribing all Sorts of Parabolas, fhot from the Elevation of any Degree in the Quadrant. Fig. 2.

Suppofe the Line AD, the furtheft Shot made from the Elevation of 45 Degrees, in Length 50 Roods, or any other equal Parts. Erect the Perpendicular AM, in Length twice AD, with AD from L, half AM defcribe the Semi-circle MNA, which divide into 90 Degrees; and AM its Diameter into 100 equal Parts.

Now from any of the faid Degrees on the Semi-circle, as 75 : 60 : 30 and 15, draw ftrait Lines to the Point A, reprefenting the Elevation of the Mortar-piece; then from A opening your Compafs to each $\frac{1}{10}$ of the Diameter

AM,

AM, describe Arches, severally to intersect the aforesaid Lines of Elevation; for from these Points of Intersection, Lines being drawn parallel to AM, till they touch the Horizon, shews thereupon the Length of the Bomb's Flight shot from its respective Elevation; as the obscure Line SR, drawn from 75 Degrees parallel to AM, touches the Horizon in R, so is AR the Length of the Shot made from the Elevation of 75 Degrees its respective Elevation, which measured on AM, will be found near 25 equal Parts.

Again, the Line VC, which from 60° is drawn parallel to AM, touches the Horizon in C, giving AC for the Length of the Shot made from the Elevation of 60 Degrees, which measured on AM, gives about $43\frac{1}{2}$; upon their obscure parallel Lines is also denoted the Height and Dimension of each Parabola.

But here it's proper to know, that a Bomb, Ball, or any other heavy Body, thrown by whatsoever Force, in falling from a certain Height in the first Moment of Time, falls 1 Space; in 2 Moments of Time, it will have fallen 4 Spaces, in 3 Moments it will have fallen 9 Spaces, and in 4 Minutes it will have fallen 16 Spaces, *&c.* from whence we conclude,

That the Fall of the Bomb is in proportion to its Weight, as the Minutes imployed in falling are to their own Squares illustrated by the annexed triangular Table, where the Point O is, from whence the Fall takes its

its Beginning; and from thence suppose the Bomb at 1 in the Table, it's clear it hath but fallen 1 Space; but being come to 2 in the Table, the Bomb hath fallen 4 Spaces; as you may count by the Number of little Triangles, contain'd in the Superfice of the great Triangle 0 2 4, or the Square of 2 is 4.

And being descended to the third Point, or fallen three Minutes, I say the Bomb hath fallen nine Spaces, as is evident by the Number of little Triangles, contain'd in the Superficies of the great Triangle 0 3 9, for the Square of 3 is 9.

Lastly, Supposing the Ball or Bomb to have fallen 4 Minutes, I say, it hath pass'd thro' 16 Spaces, or equal Distances, as clearly appears by the 16 little Triangles, making up the Superficies of the great Triangle 0 4 16: also the Square of 4 is 16.

The Geometrical Projection.

And first to describe the Parabolas shot from the Elevation of 75, and 15 Degrees, its Complement to 90.

From all the Intersections of the diametrical Arches upon RS, the Line of the given Elevation, draw obscure Lines parallel to the Diameter AM, on which are set off all the several Distances contained in the aforesaid triangular Table, the first whereof being 1.

With

With your Compasses take 1 from the Diameter A M, and lay it on the first parallel Line, and mark it 1, and that gives the Space or Length of the Fall in the first Moment of Time.

Again, in the Table find the 2 d Min. and you will have 4 for its Square; which take from the Scale A M, and set upon the next Parallel, marking it with the Figure of 4.

Then in the Table find 3 Minutes, and you'll have 9 for its Square; wherefore take 9 from your Scale A M, and lay it upon the third Parallel.

In like manner, for the fourth Minute, you will find 16 its Square, which taken from A M, lay it upon the fourth Parallel.

Now for the fifth Minute, you will find 25 its Square, therefore from your Scale A M, taking 25, set it off upon the fifth Parallel, and write upon it 25; and continuing this manner of Operation, till you come to the *9th* Minute of Time, whose Square is 81; you will have got 9 several Points as marked in the Figure, 1, 2, 3, 4, 5, 6, 7, 8, 9; all which being traced from point to point with a firm Hand, will describe the Parabolick Shot from the Elevation of 75 Degrees; whose Horizontal Length is A R, equal to S T, which measured on A M, gives near 25 equal Parts; and in this manner proceed with the next Parabola A E C, shot from the Elevation of 60 Degrees; whose Horizontal Distance

A C,

AC, equal to VX, is found on the Scale AM, to be in Length $43\frac{1}{2}$ equal Parts.

And the like for the other three remaining Parabolas, *viz.* AFD: AGC: and AHR: and it's done.

FINIS.

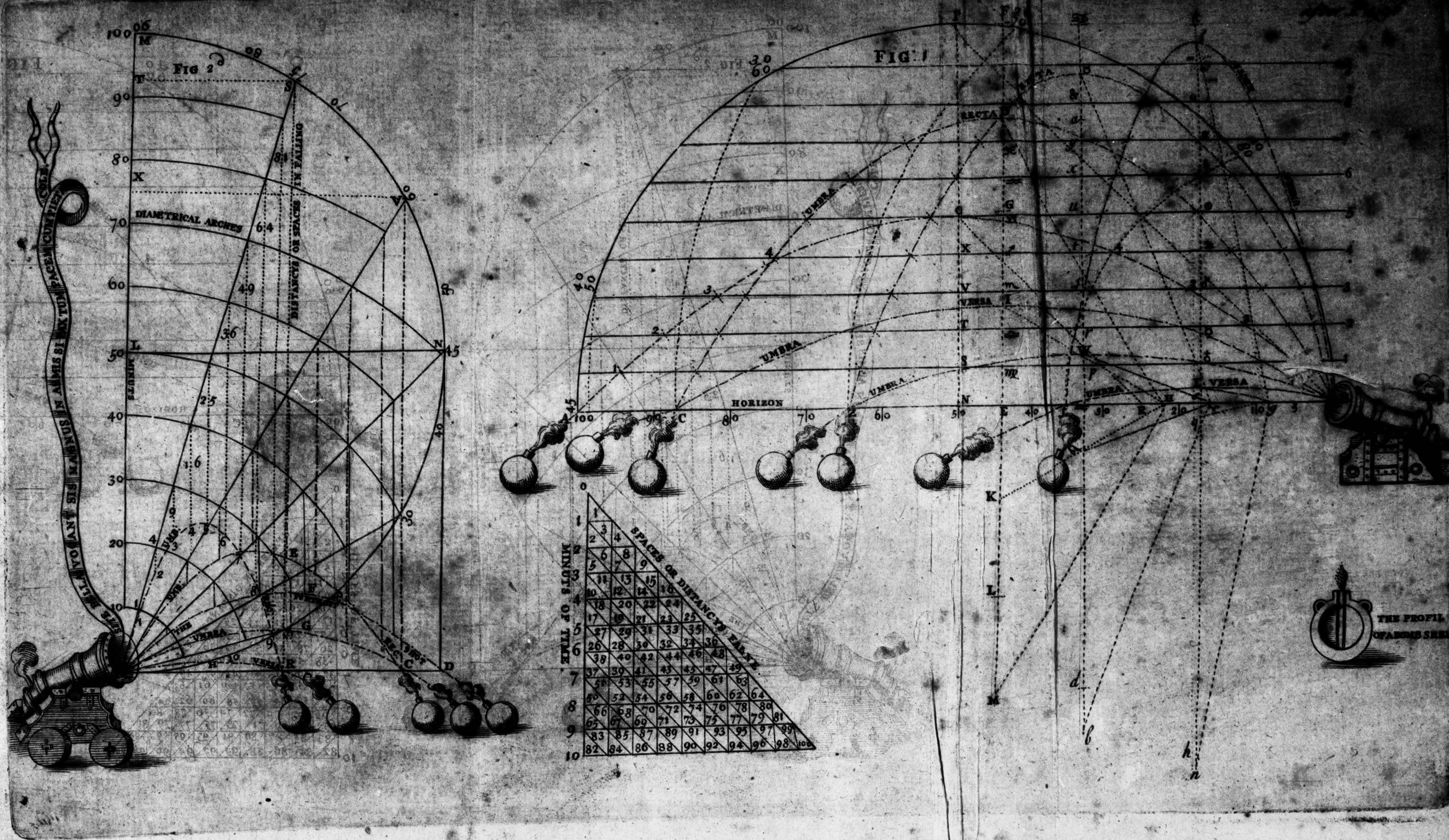


Fig 2
FIG 1
DIAMETRICAL ARCHES
MINUTS
HORIZON
UMBRA
UMBRA VERSA
RECTA
SPACES OR DISTANCYS FALNE
MINUTS OF TIME
THE PROFIL OF A BOMB SHEL